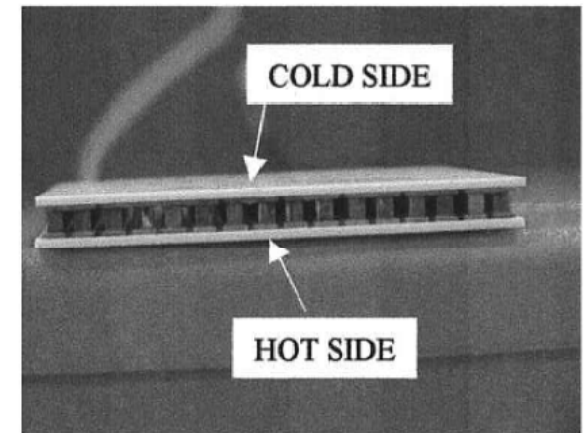
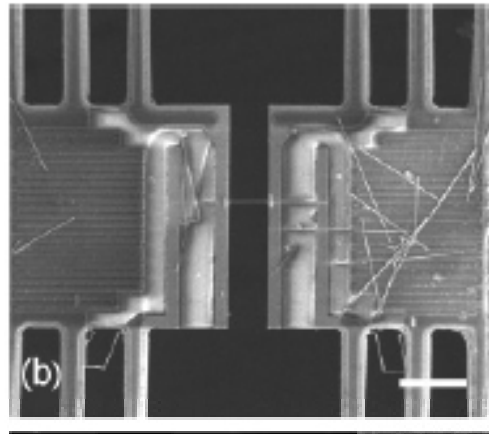
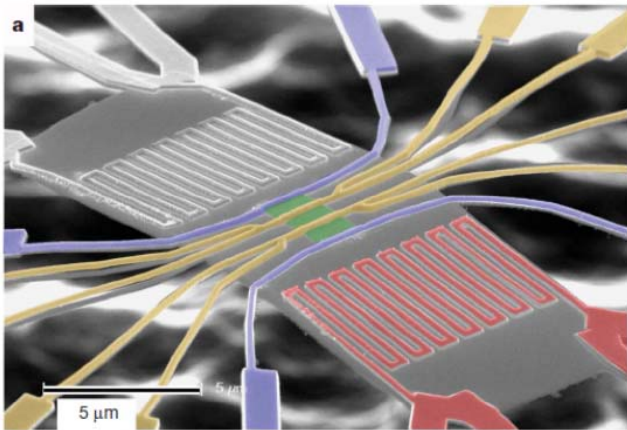
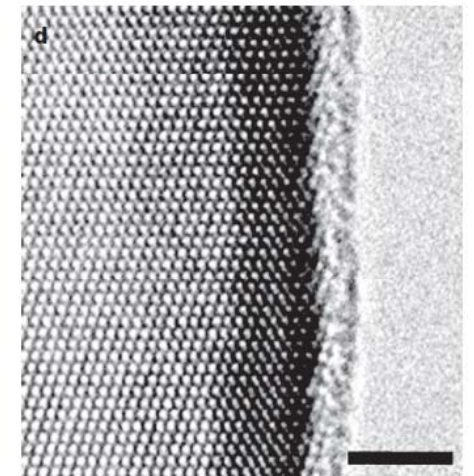
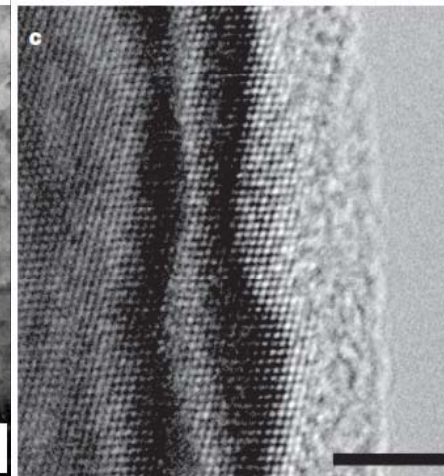
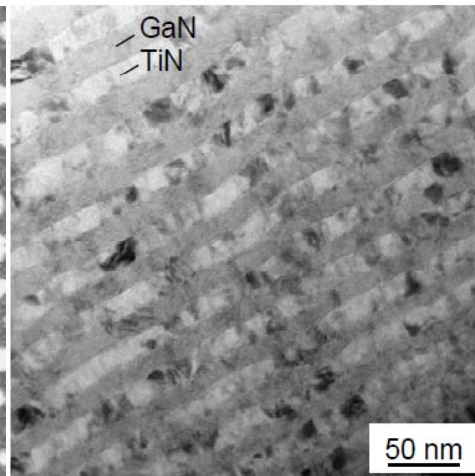
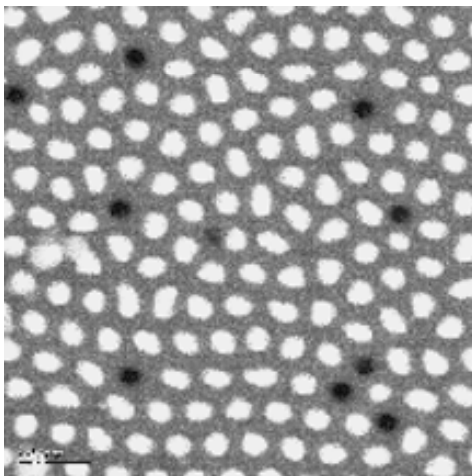




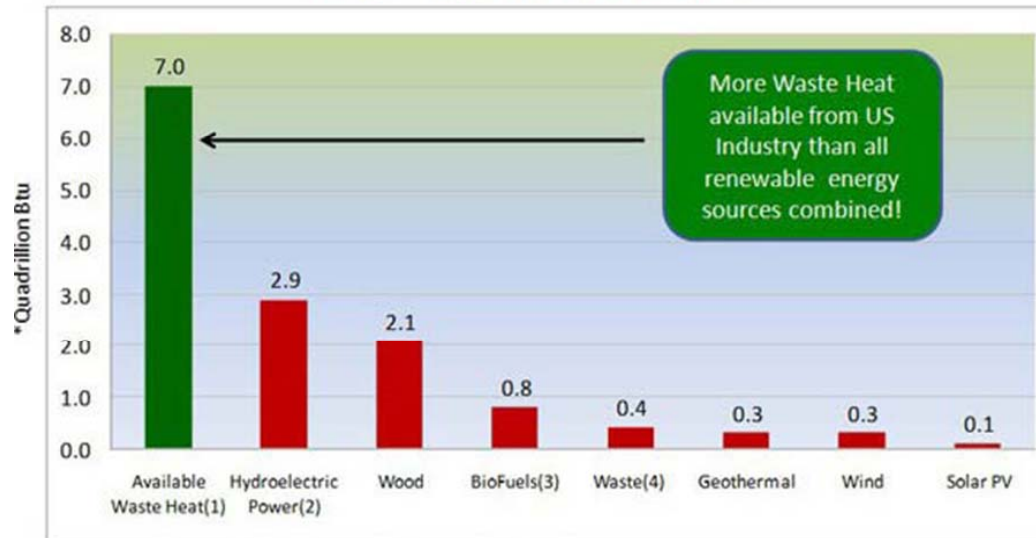
Nanomaterials for Thermoelectric Applications



Renewable Energy

Renewable Energy Consumption by Source

US DOE - EIA Annual Energy Survey 2006



*One Quadrillion Btu is equal to one US football field 3.5 miles high of oil

1) 24.7 Quads of energy is used by industry – of this 20-50 percent is lost in the form of Waste Heat (US DOE)

2) Conventional hydroelectric power.

3) Fuel ethanol and biodiesel consumption, plus losses and co-products from the production of fuel ethanol and biodiesel.

4) Municipal solid waste from biogenic sources, landfill gas, sludge waste, agricultural byproducts, and other biomass. Through 2000, also includes non-renewable waste (municipal solid waste from non-biogenic sources, and tire-derived fuels).



Solar



Wind



Water



Biomass



Geothermal

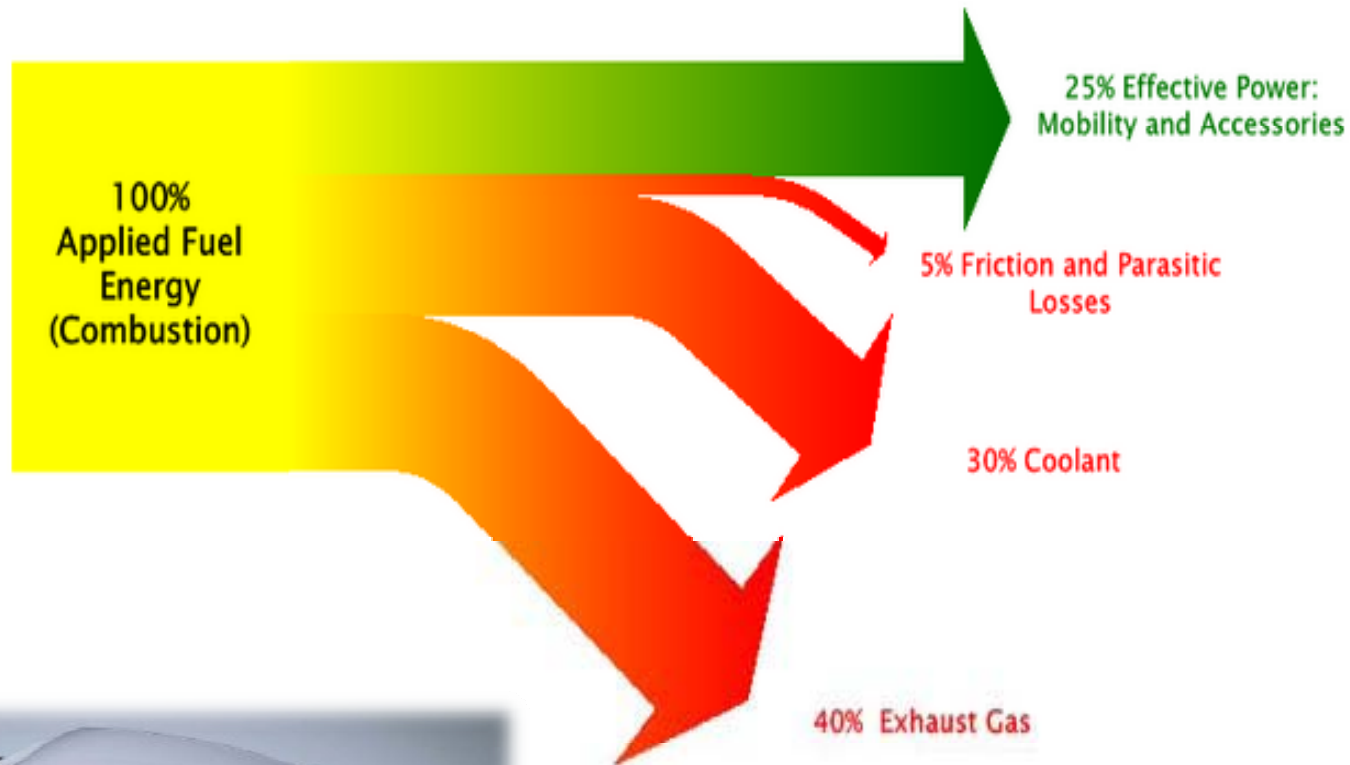


Hydrogen & Fuel Cells



Exhaust Waste Heat to Electrical Power

Typical Energy Split in Gasoline Internal Combustion Engines



GM has 350Watt avg systems on a truck can get 3% to 4% fuel saving with better integration.

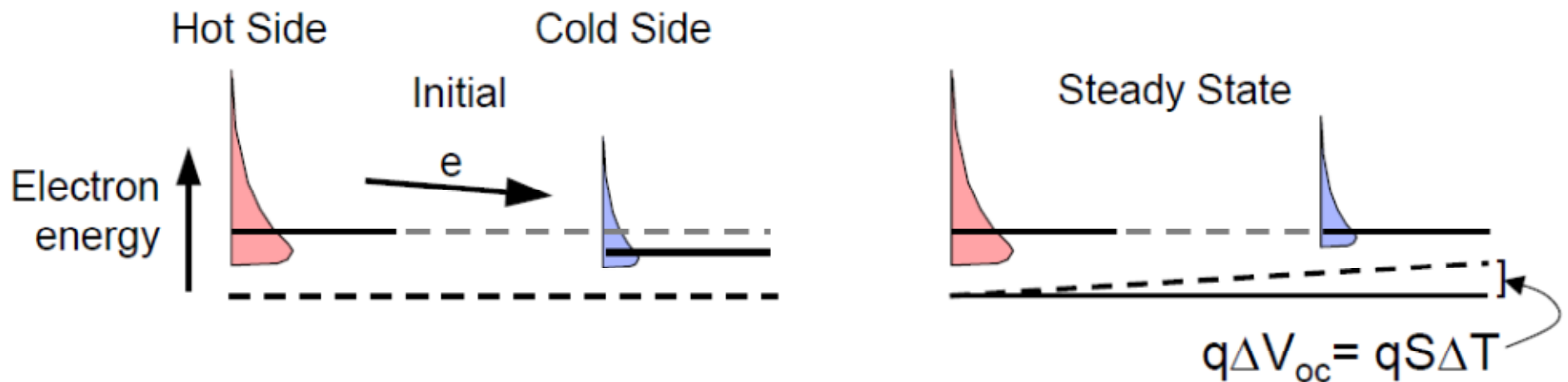
The Seebeck Effect (1821)

The voltage difference, V , produced across the terminals of an open circuit made from a pair of dissimilar metals.

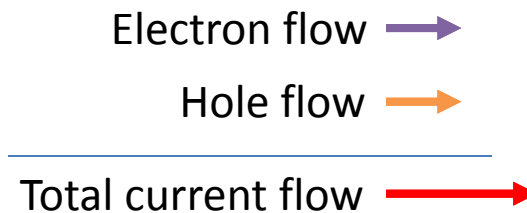
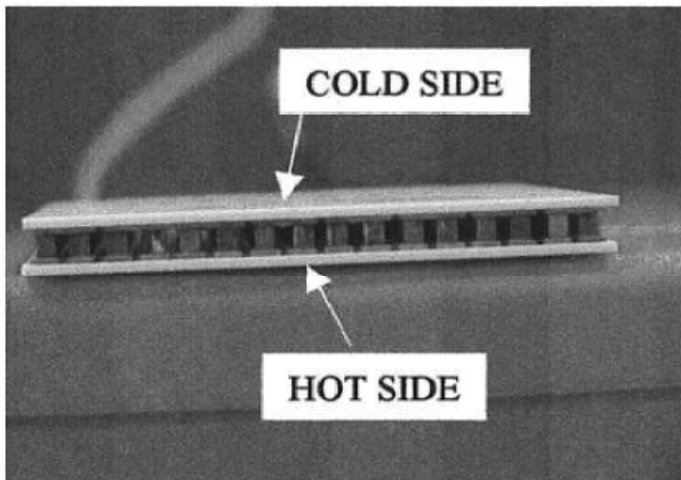
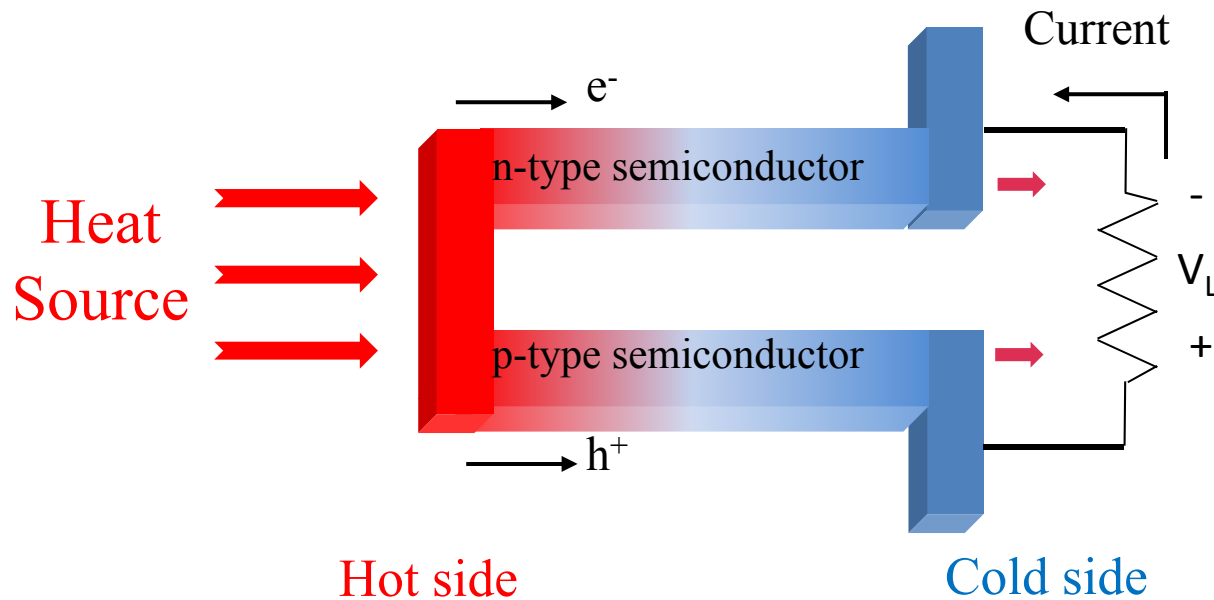
- A. Initially, carrier (electrons or holes) move from hot to cold
- B. Resulting potential difference opposes further current flow
- C. The open circuit voltage is proportional to ΔT .

$$\lim_{\Delta T \rightarrow 0} \{\Delta V\} = S\Delta T$$

where S is the Seebeck coefficient [V/K]

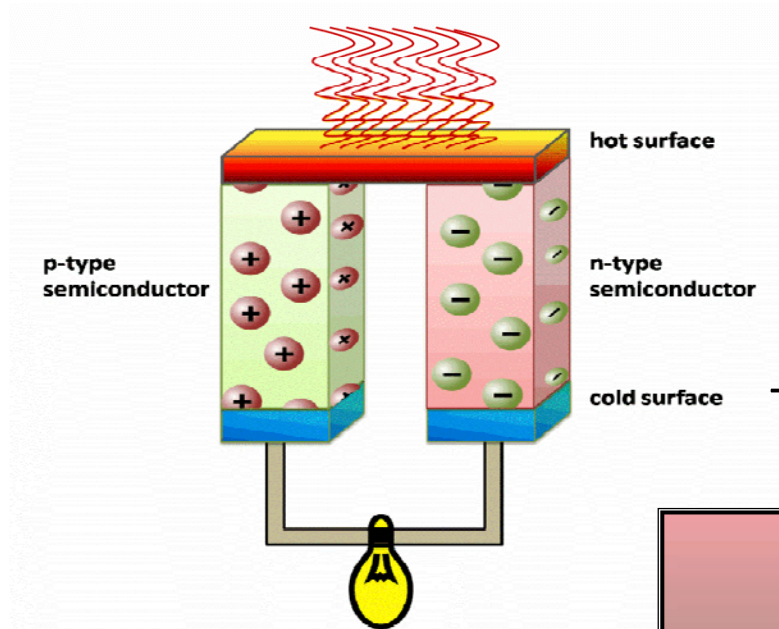


TE Generator Working Principle



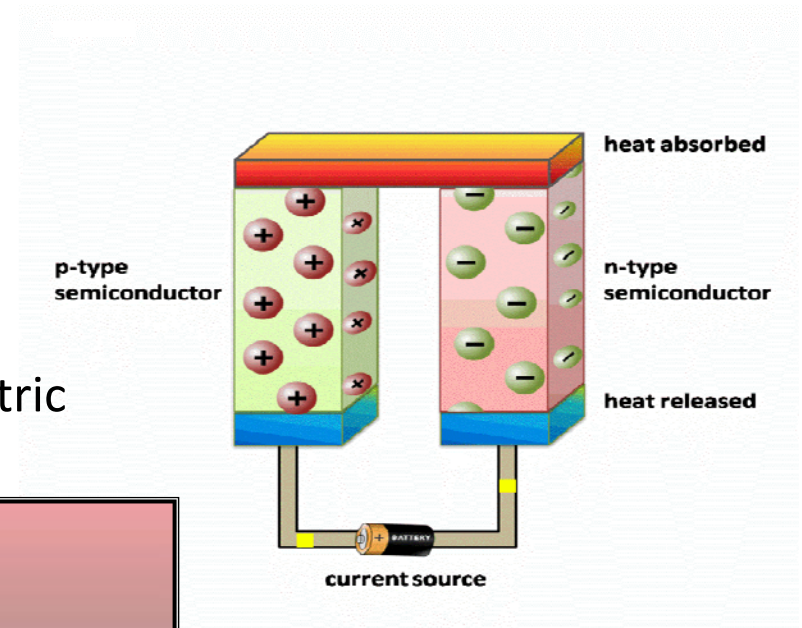
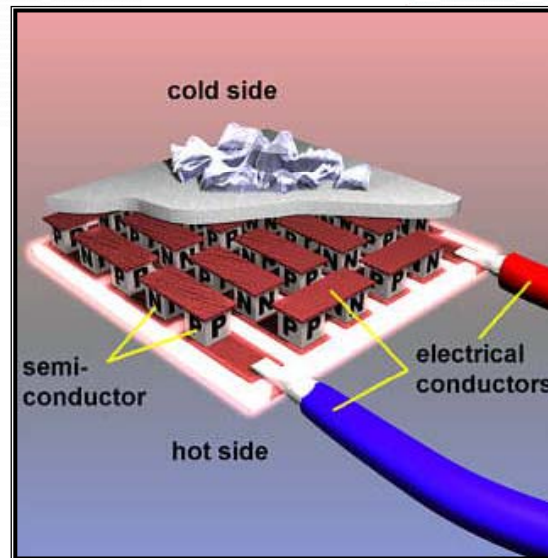
TE Generator Working Principle

The Seebeck and the Peltier Effect



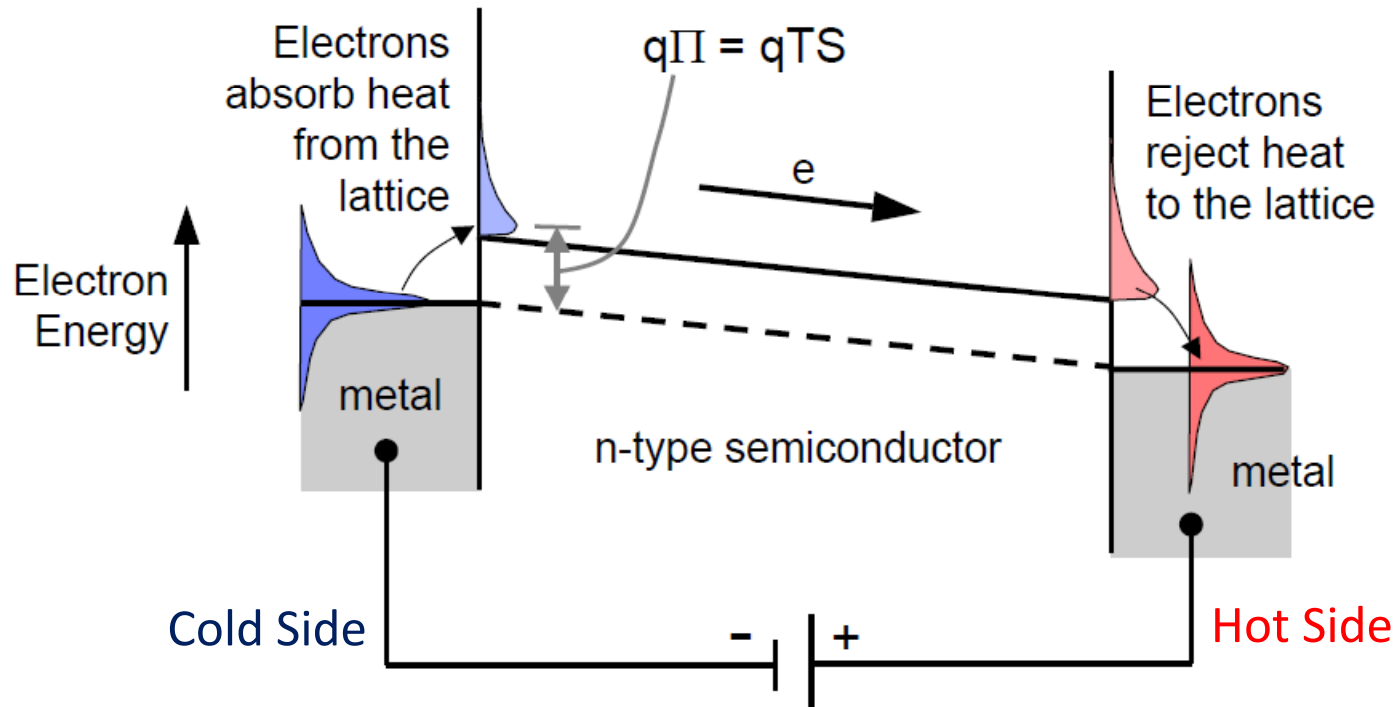
Thermoelectric
Power Generation

Thermoelectric Device



Thermoelectric
Cooling

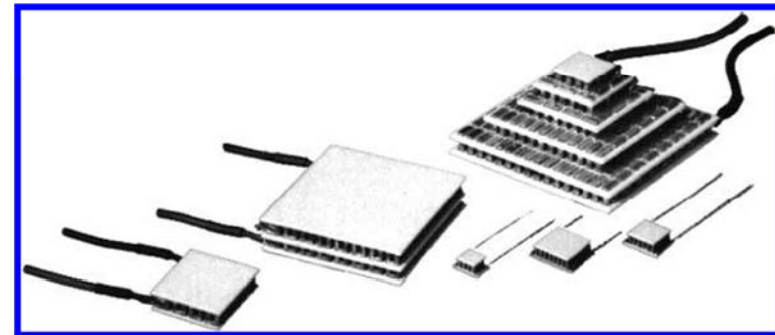
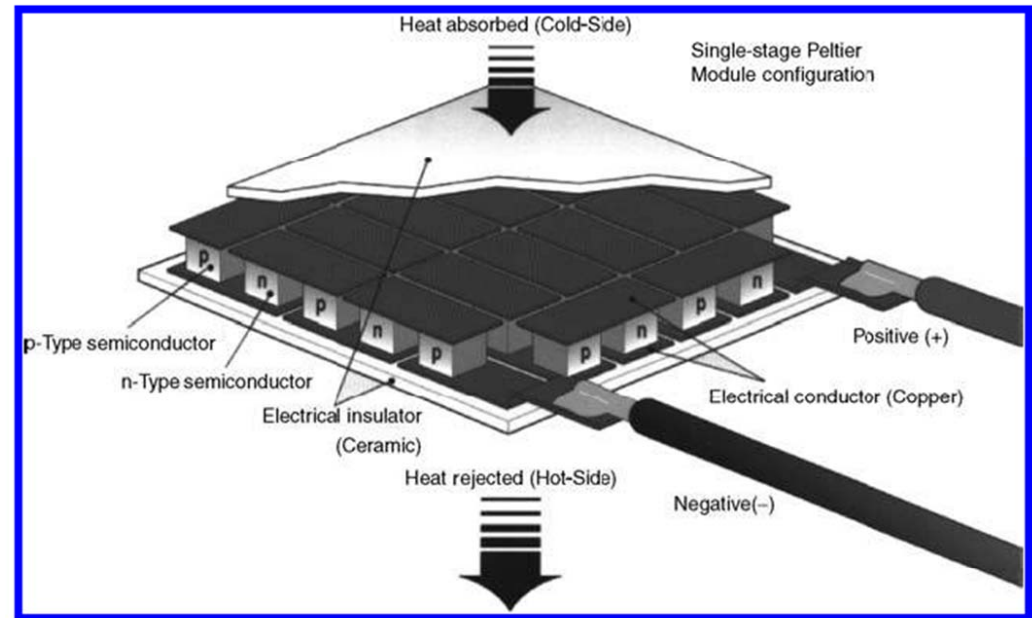
The Peltier Effect (1834)



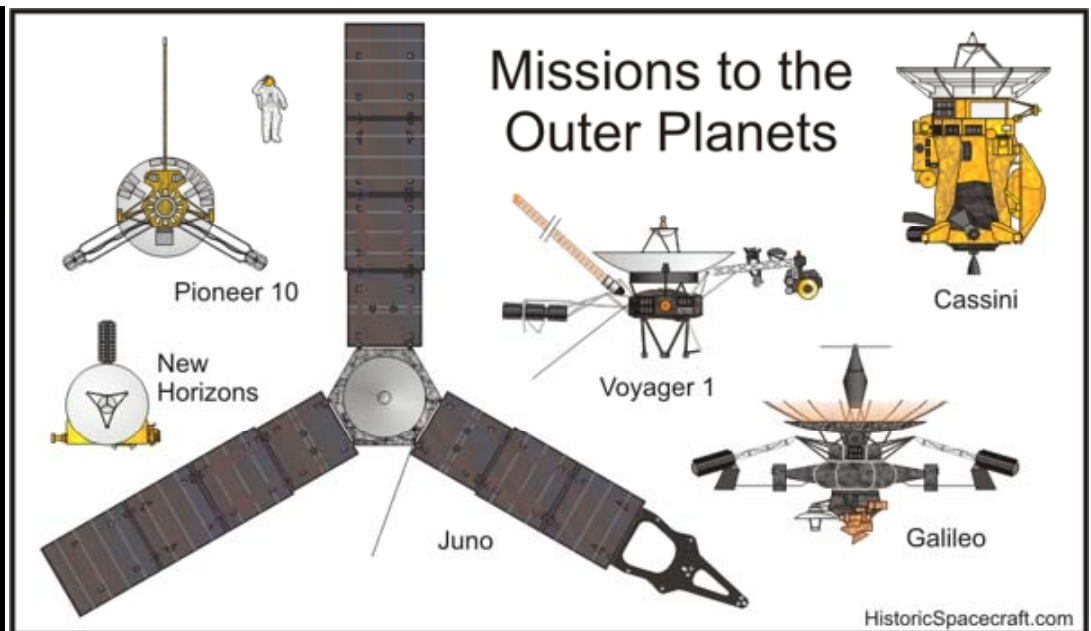
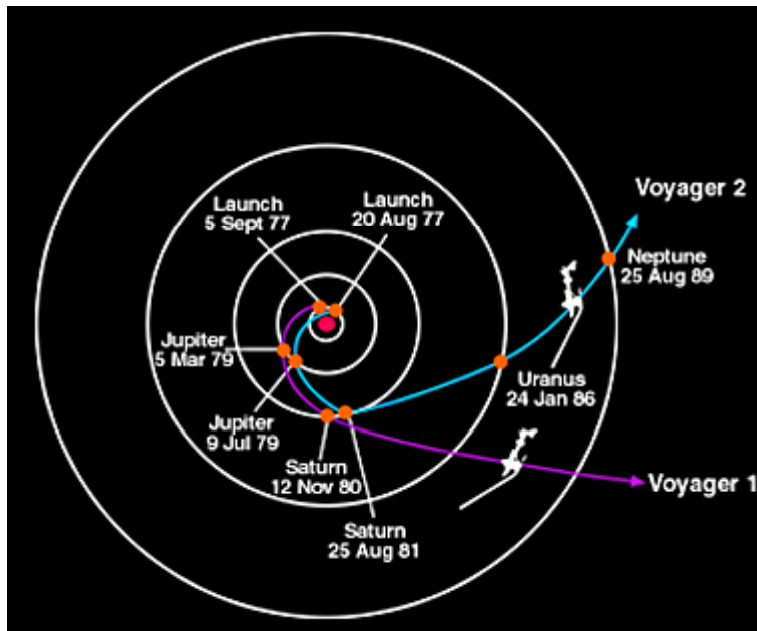
Driven by an applied potential, electrons (holes) absorb heat from the lattice at the cold side, and reject heat to the lattice at the hot side
→Refrigeration.

Benefits of Thermoelectric devices

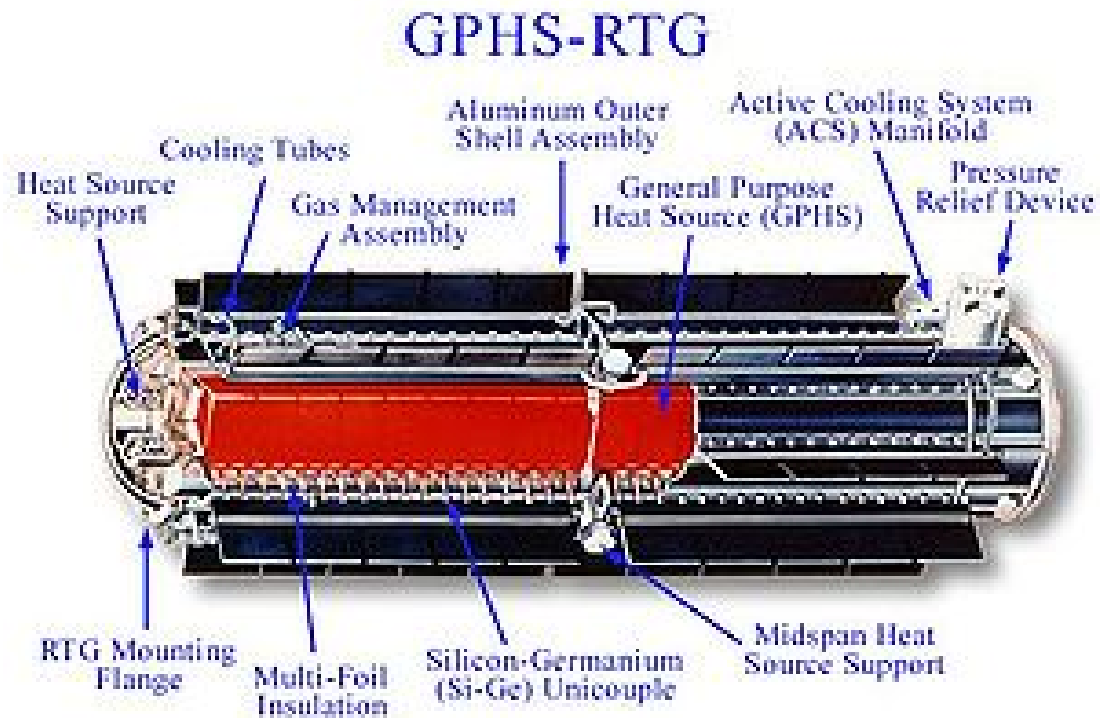
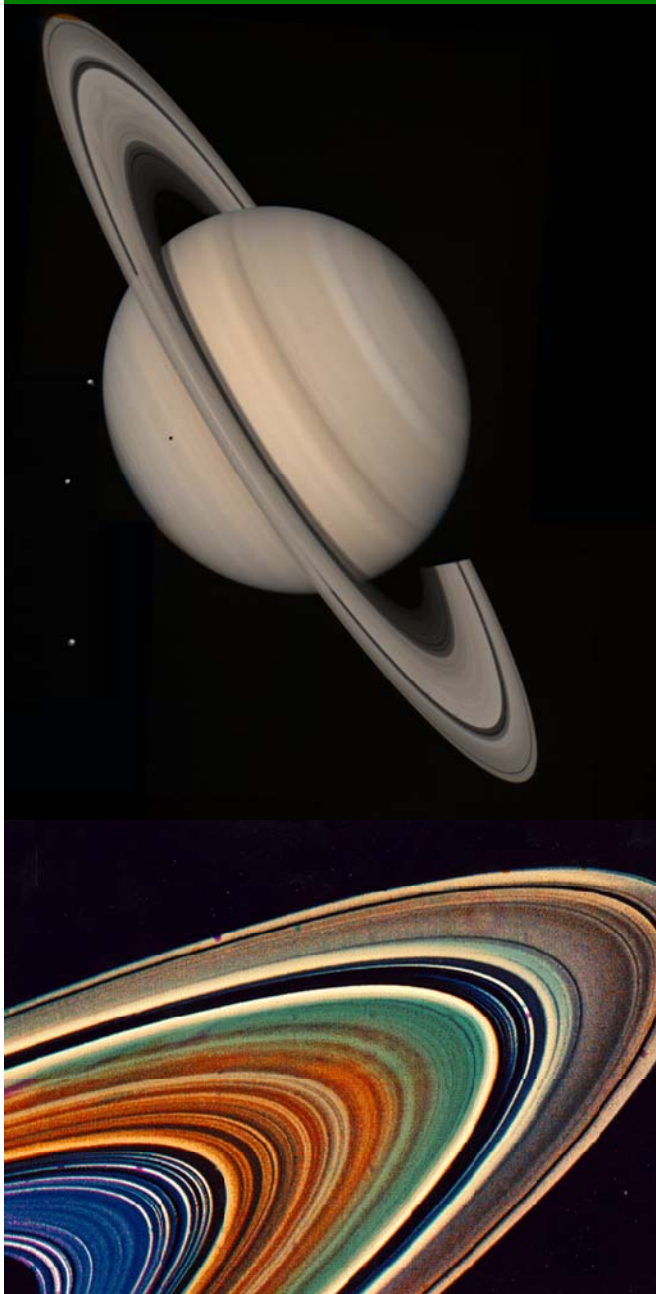
- ✓ No moving elements
- ✓ No working fluids and gases
- ✓ Low-noise operation
- ✓ Reduced size and weight
- ✓ High reliability — KRYOTHERM guarantees lifetimes of more than 200,000 hours for our TEDs
- ✓ Easy switching from cooling to heating mode



Example of TE Devices: Voyagers

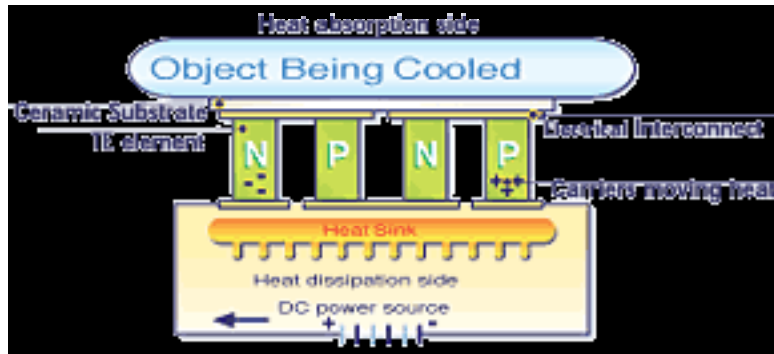


Example of TE Devices: RTG



Radioisotope thermoelectric generator (RTG, RITEG)

Example of TE Devices: CCD Cooling



Koolatron P-27 Voyager - A Thermoelectric Cooler - 12 Volt Cooler (\$139.95)



Thermoelectric Efficiency and the Figure-of-Merit

The efficiency of the generator is given by

$$\phi = \frac{\text{energy supplied to the load}}{\text{heat energy absorbed at hot junction}}$$

q is the heat input required at the hot junction. Consider three effects:

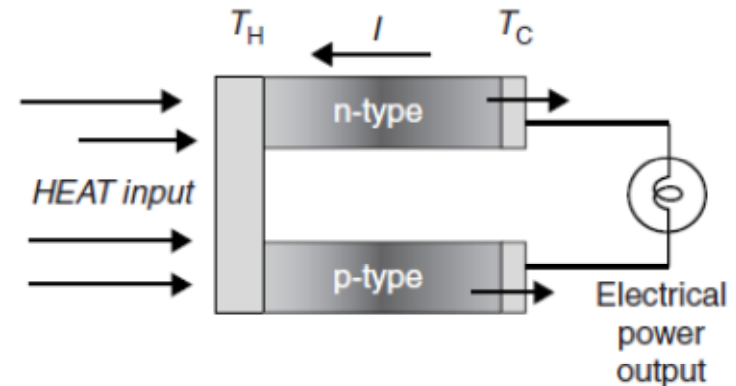
1. Peltier heat $q_1 = -\pi I$
2. A portion of the Joule heat (It is assumed that both elements are of equal length and that element b has unit cross-sectional area.)

$$q_2 = -\left(\frac{I^2}{2}\right)[\rho_b + (\rho_a/A)]L$$

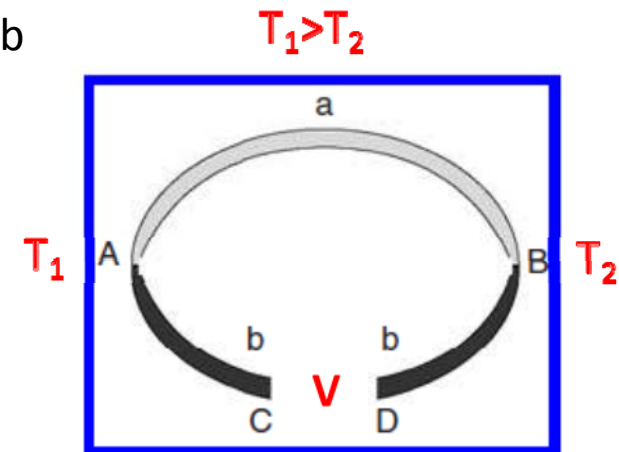
3. Heat conducted away from the hot junction by thermal conduction

$$q_3 = (\kappa_b + A\kappa_a)\Delta T/L$$

$$(q_H = -\kappa \nabla T)$$



The Seebeck effect (Thermoelectric generation)



Schematic basic thermocouple.

$$\eta = \frac{I^2 R_L}{\left\{ (\kappa_b + \kappa_a A) \left(\frac{\Delta T}{L} \right) - \left(\frac{I^2}{2} \right) \left[\rho_b + \left(\frac{\rho_a}{A} \right) \right] L - \pi I \right\}}$$

Where $I = \frac{V_L}{R} - \frac{S_{ab} \Delta T}{\left\{ R_L + \left[\rho_b + \frac{\rho_a}{L} \right] L \right\}}$

$$\eta = \left(\frac{\Delta T}{T_H} \right) \left[\frac{\sqrt{1 + Z\bar{T}} - 1}{\sqrt{1 + Z\bar{T}} + \frac{T_c}{T_h}} \right]$$

The efficiency of the device is limited by the Carnot efficiency ($\Delta T/T_H$)

and $\bar{T} = \frac{T_H + T_L}{2}$

and $Z = \frac{S^2}{\left[\sqrt{\frac{\kappa_b}{\rho_b}} + \sqrt{\frac{\kappa_b}{\rho_b}} \right]^2}$ is called the **Figure of merit**.

In practice, the two arms of the junction have similar material constants, in which case the concept of a figure-of-merit for a material is employed and given by

$$Z = \frac{S^2 \sigma}{\kappa}$$

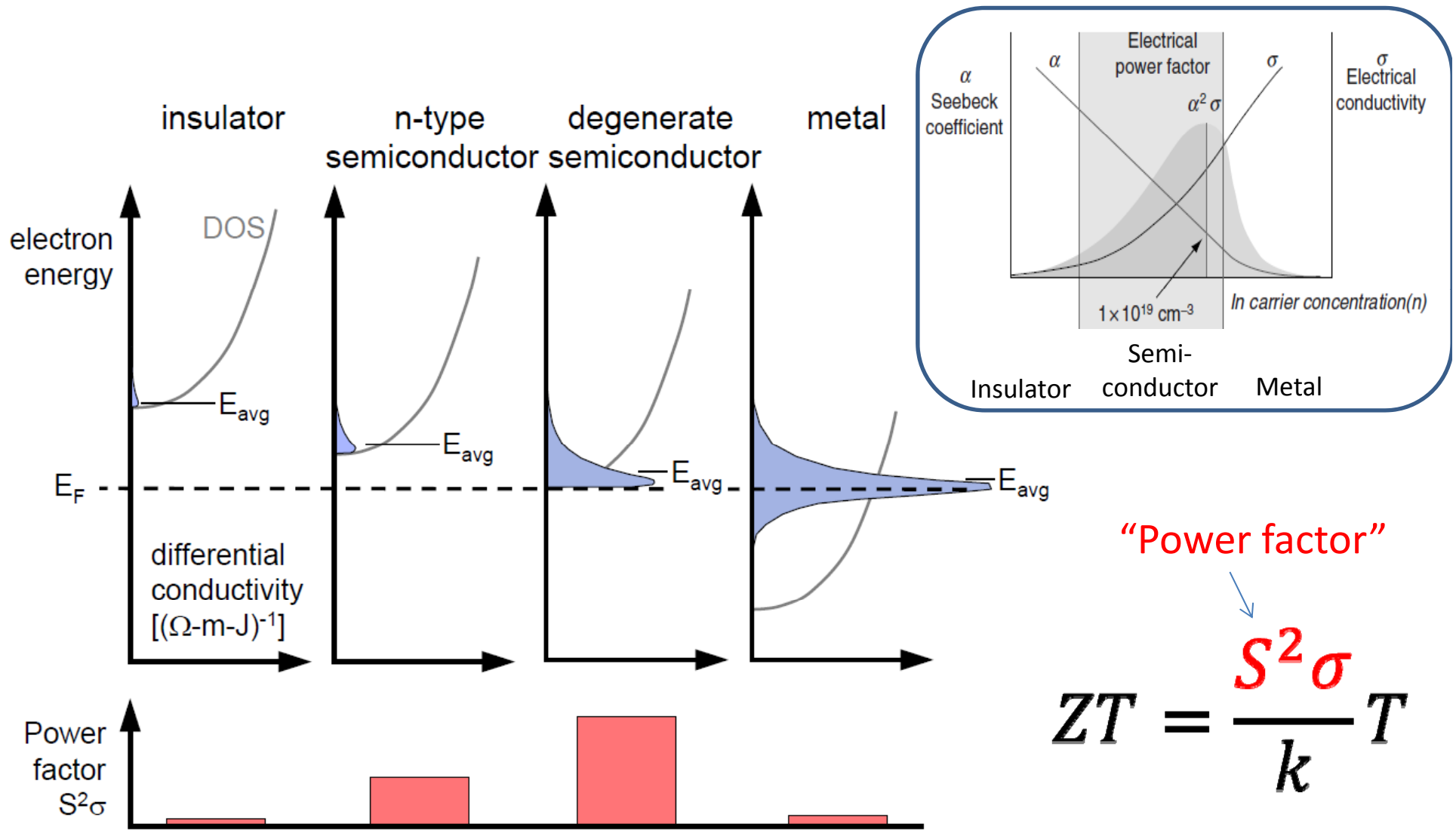
S : Seebeck coefficient

T : absolute temperature

σ : electrical conductivity

κ : thermal conductivity

Differential Electron Conductivity



Selection of Thermoelectric Materials

$$Z = \frac{S^2 \sigma}{\kappa}$$

Electrical conductivity σ
Thermal conductivity κ
Seebeck coefficient S

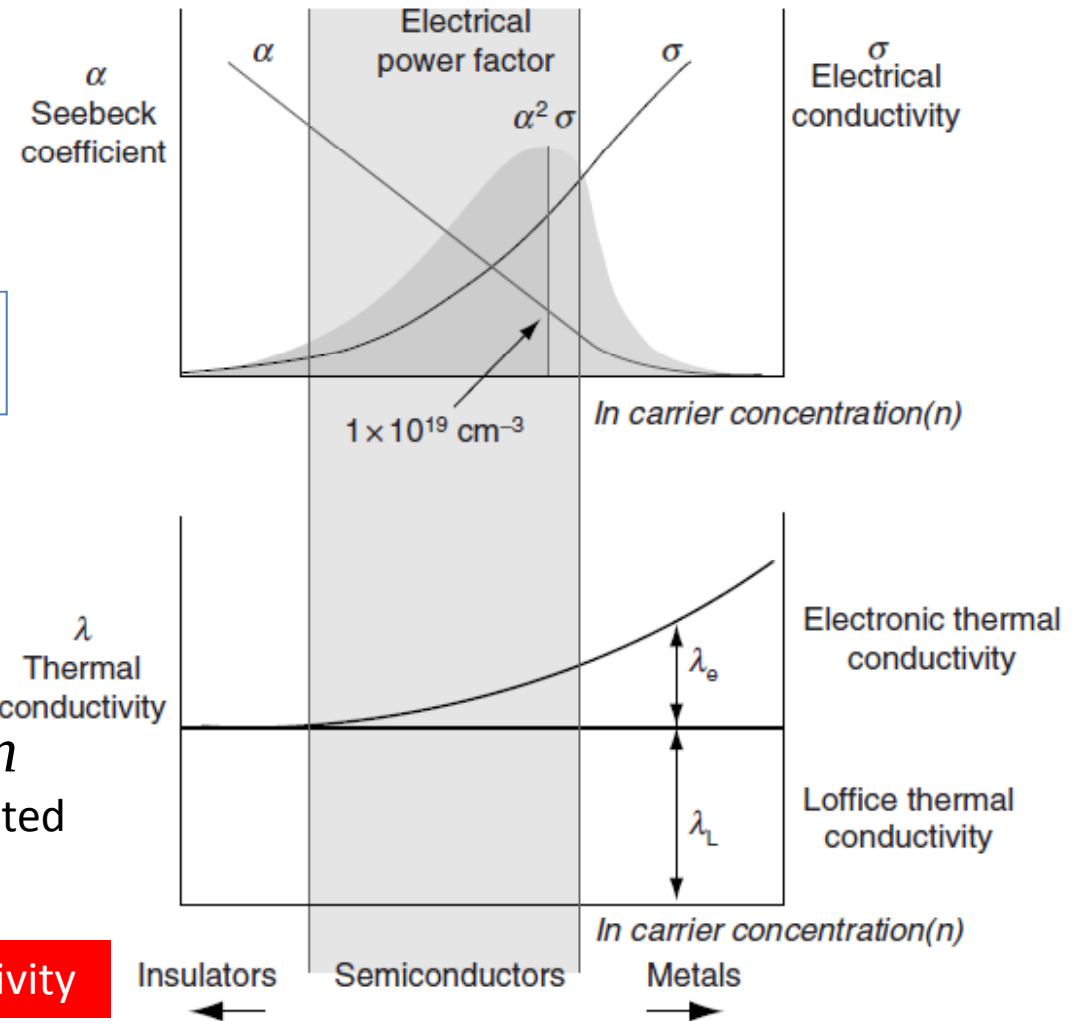
Free carrier
density n

The thermal conductivity

$$\kappa = \kappa_{\text{Lattice}} + \kappa_{\text{Electron}}$$

Lattice part is roughly independent of n
while the electronic part is directly related
to the electrical conductivity

High power factor, Low thermal conductivity

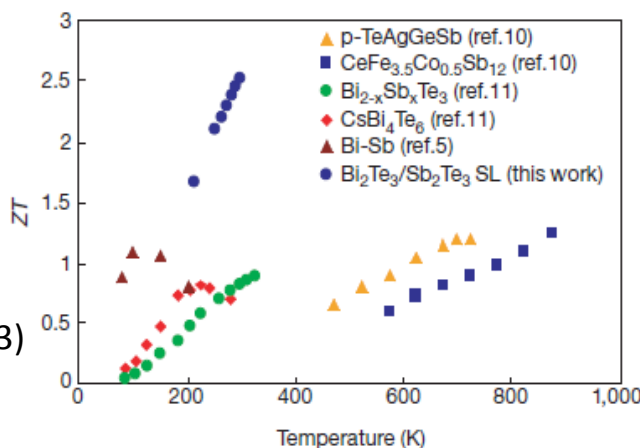


Schematic dependence of electrical conductivity, Seebeck coefficient, power factor, and thermal conductivity on concentration of free carriers.

ZT in This Decade

Year	Journal	Materials	ZT	T(K)	Institute
2011	Nature	PbTe _{1-x} Se _x (Buck)	1.8	850K	MS, California Institute of Technology
2009	Nature	In ₄ Se _{3-d} (Buck)	1.5	700K	Samsung
2008	Nature	Si (52nm rough NWs)	0.6	300K	Berkeley
2008	Nature	Si (20nm NWs)	1.0	200K	CE, California Institute of Technology
2008	Science	BiSeTe (buck)	1.5	400K	Physics, Boston College
2004	Science	AgPb _m SbTe _{2+m} (buck)	2.1	800K	Michigan State University
2003	Nature	Bi ₂ Te ₃ /Pb ₂ Te ₃ SL	2.6	300K	Research Triangle Institute
2002	Science	PbSeTe/PbTe SL	1.6	300K	Massachusetts Institute of Technology
Before			~1		

Y. Z Pei et al, Nature 473, 66 (2011)
 J. S. Rhyee et al, Nature 459, 965 (2009)
 A. I. Hochbaum et al, Nature 451, 163 (2008)
 A. I. Boukai et al, Nature 451, 168 (2008)
 B. Poudel et al, Science 320, 634 (2008)
 K. F. Hsu et al, Science 303, 818 (2004)
 R. Venkatasubramanian et al, Nature 413 597 (2003)
 T. C. Harman et al, Science 297, 2229 (2002)



Nature Materials	Publication
2011	4
2010	4
2003-2009	13

ZT & Efficiency

Efficiency η

$$\eta = \frac{T_H - T_C}{T_H} \frac{\sqrt{1 + ZT} - 1}{\sqrt{1 + ZT} + T_C / T_H}$$

Figure-of-merit ZT

$$ZT = T \frac{S^2 \sigma}{\kappa}$$

Thermal conductivity κ

$$\kappa = \kappa_L + \kappa_{el}$$

Efficiency examples

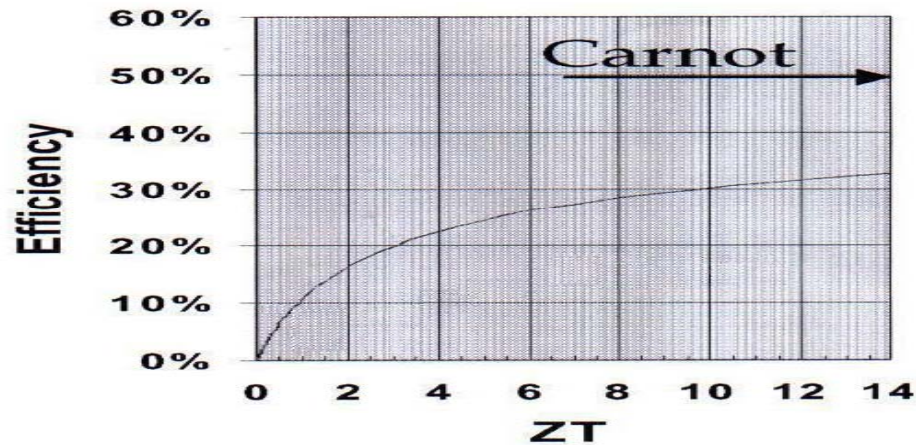
ZT	T_H/K	T_C/K	η
0.5	673	373	5.6%
1	673	373	9.4%
2	673	373	14.3%
3	673	373	17.4%
3	773	373	20.8%
3	873	373	23.6%

Thermal conductivity κ

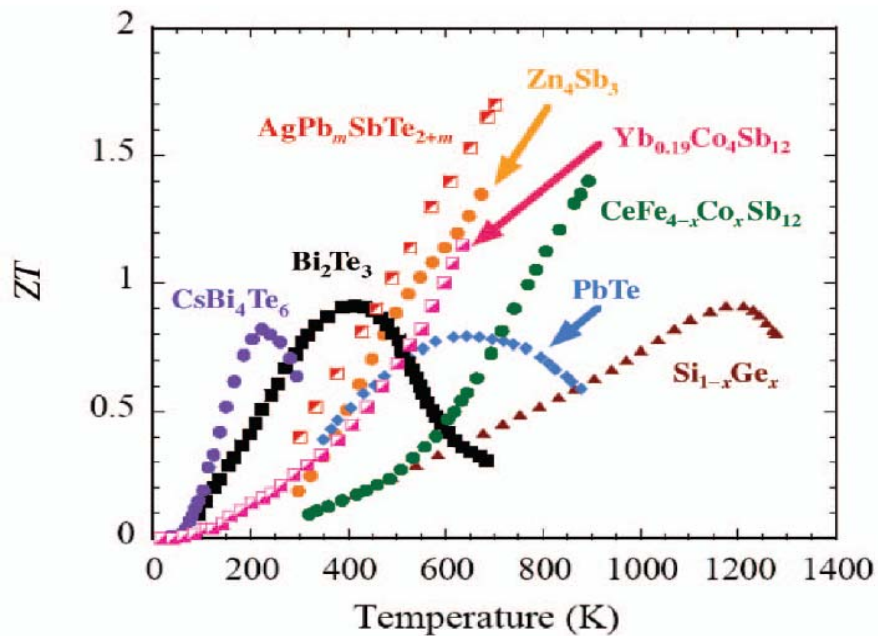
$$\kappa_{el} = L_0 \sigma T$$

- To match a refrigerator, an effective $ZT=4$ is needed
- To efficiently recover waste heat from car, $ZT=2$ is needed

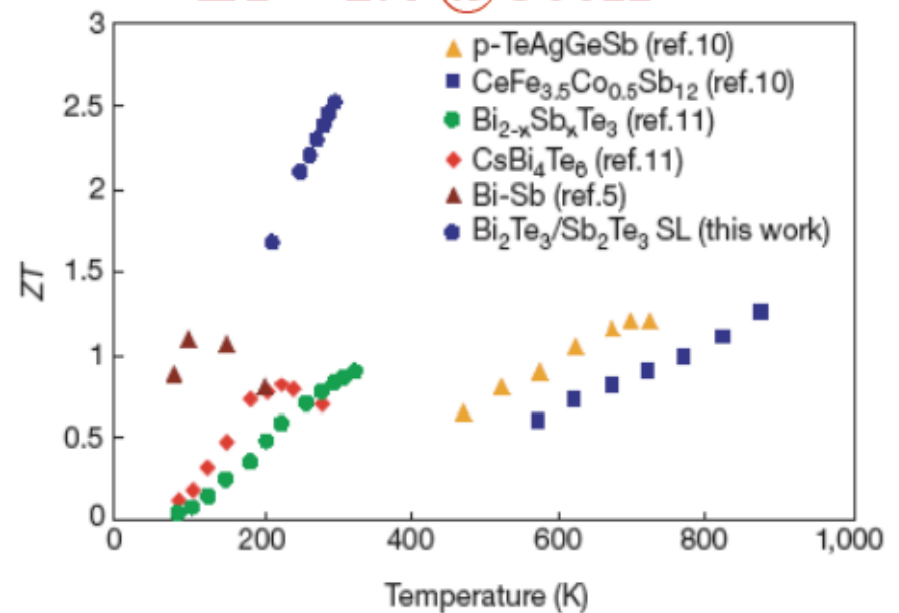
ZT & Efficiency



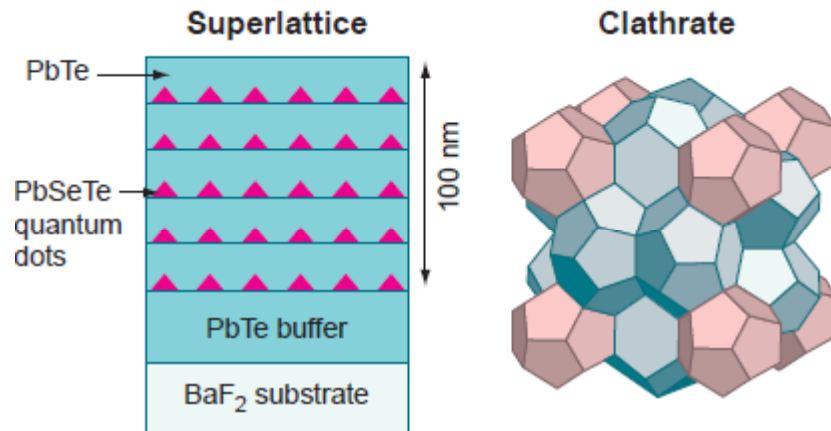
Efficiency for $T_c / T_h = 0.5$



$ZT > 2.4 @ 300\text{K}$



TE Device with Superlattice Structure



Promising new materials. (Left) Nanoengineered thermoelectric materials of this kind are prepared with molecular beam epitaxy. (Right) In this model clathrate crystal structure, the cubic crystal structure is composed of two types of polyhedra that consist of clusters of 20 or 24 atoms. In self-assembled structures of this type, nature does the nanoengineering.

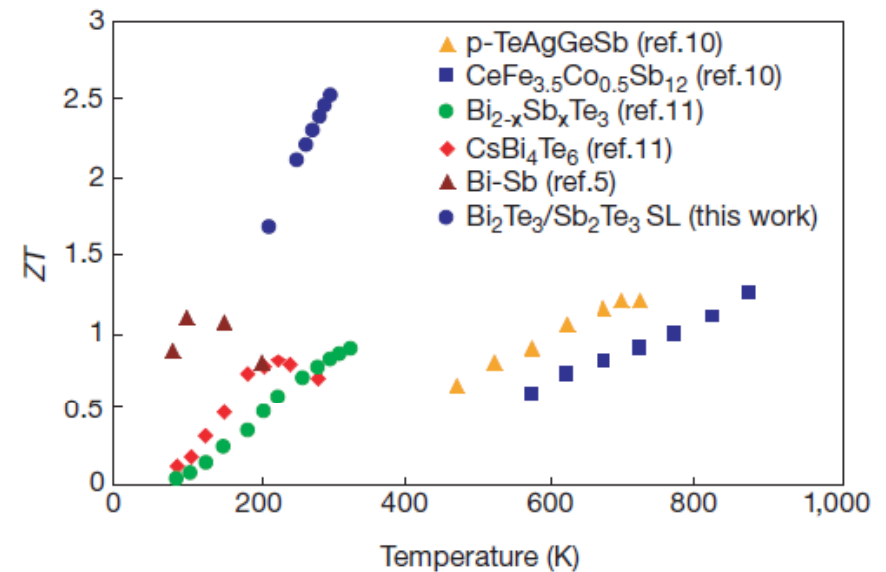
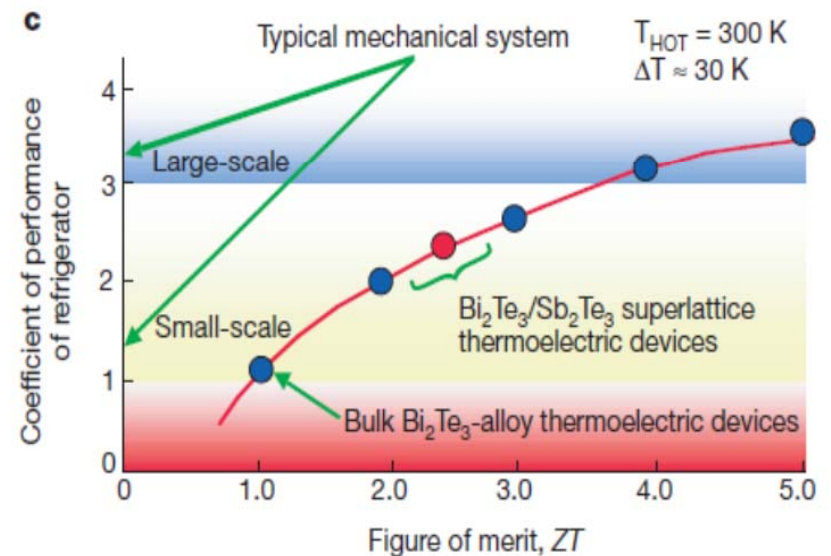


Table 1 Theoretical and experimental lattice thermal conductivities

Material	Thermal conductivity (W m ⁻¹ K ⁻¹)
$K_{\min}$ of Bi ₂ Te ₃ (a-b axis), Slack model ³⁴	0.55
$K_{\min}$ of Bi ₂ Te ₃ (c axis), Slack model ³⁴	0.28
$K_{\min}$ of Bi ₂ Te ₃ (a-b axis), Cahill model ³⁵	0.28
$K_{\min}$ of Bi ₂ Te ₃ (c axis), Cahill model ³⁵	0.14
K_L of Bi _{2-x} Sb _x Te ₃ alloy (a-b axis)	0.97
K_L of Bi _{2-x} Sb _x Te ₃ alloy (c axis)	0.49
K_L of Bi ₂ Te ₃ /Sb ₂ Te ₃ superlattice (c axis)	0.22

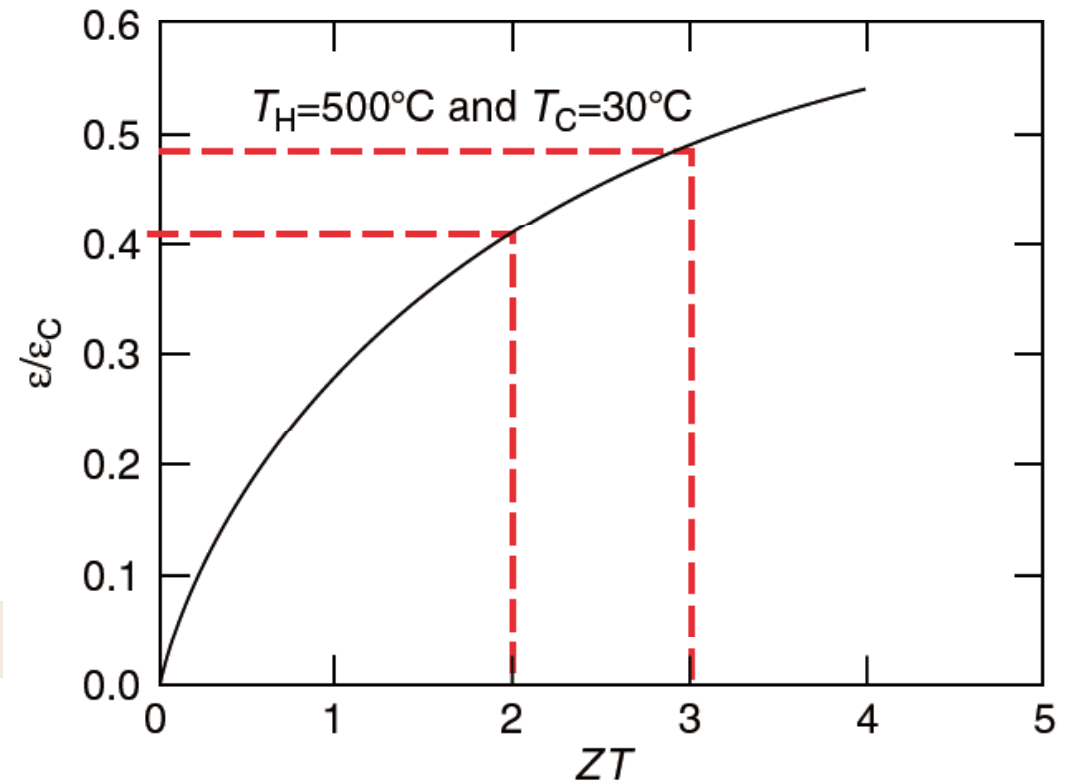
Lattice thermal conductivity (K_L) of the Bi₂Te₃/Sb₂Te₃ superlattice (period ~ 50 Å) compared with K_L observed in the respective alloys and the theoretical minimum lattice thermal conductivity ($K_{\min}$) from various models.



Efficiency and ZT

$$\eta_{TE} = \eta_C \left(\frac{\sqrt{1 + ZT} - 1}{\sqrt{1 + ZT} + \frac{T_C}{T_H}} \right)$$

Carnot efficiency, $\eta_C = (T_H - T_C)/T_H$

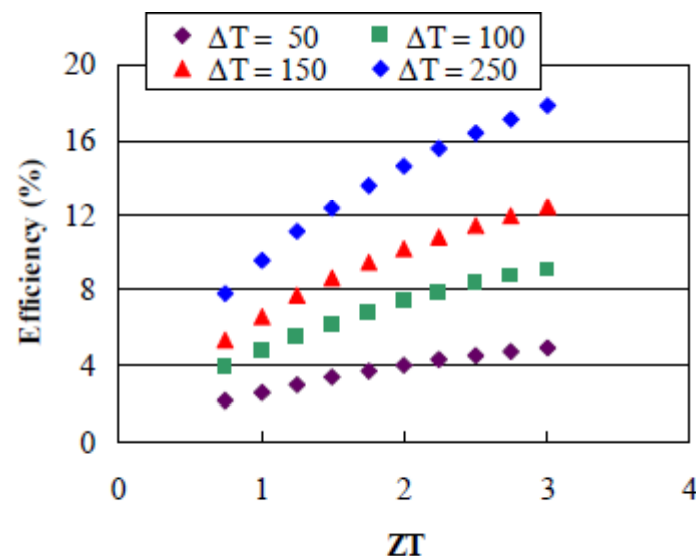
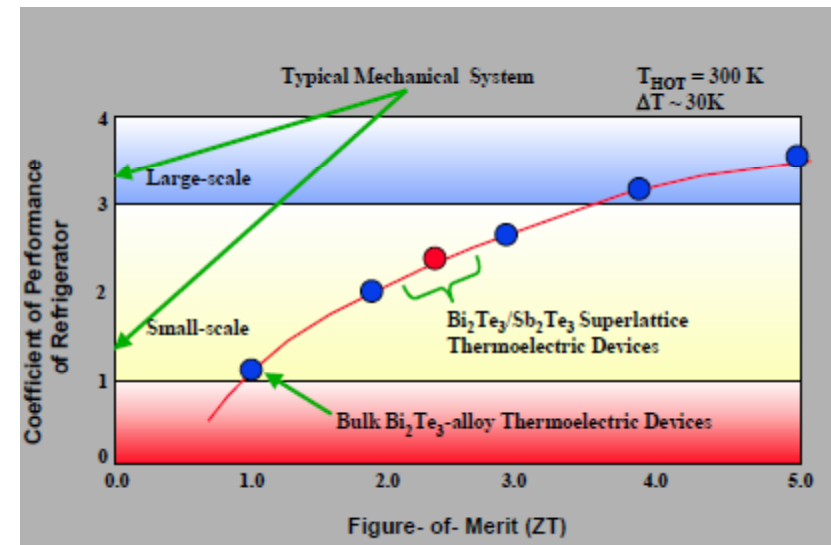
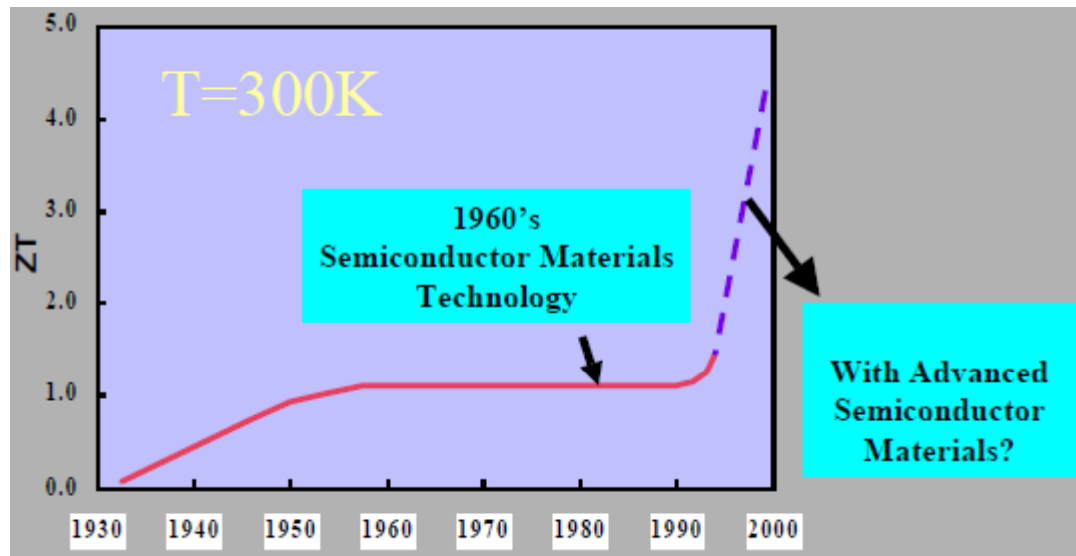


Searching for New Thermoelectric Materials

- Figure of Merit = $ZT = (\alpha^2\sigma/k)T$
 - T = Absolute Temperature
 - α^2 = (Seebeck coefficient)²
 - ❖ Tells how much average thermal energy is transported by each carrier
 - σ = electrical conductivity
 - ❖ Tells how much the carriers can transport energy without Joule loss
 - k = thermal conductivity
 - ❖ Tells how small is the reverse flow of heat from the cold-side to the hot-side, opposing the electron-transport of heat
- Minimize thermal conductivity and maximize electrical conductivity

Can nanotechnology be adopted to minimize thermal conductivity and maximize electrical conductivity?

Searching for New Thermoelectric Materials



- ZT need to improve over 1.3 at 300K for a major impact in electronics cooling and around 2.5 for a revolutionary impact in air-conditioning, and power from waste-heat

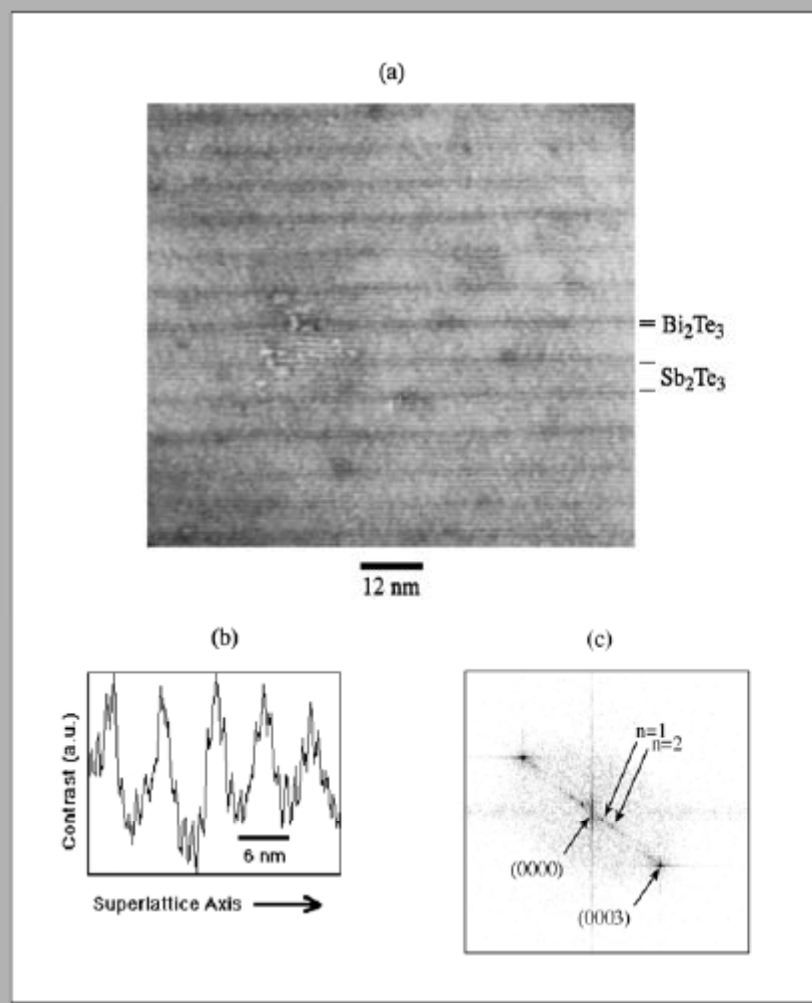
New Approaches

- New Bulk Materials
 - Skutterudites (Rensselaer, Oak Ridge, JPL, 1992)
 - ❖ Cage-structures with rattling atoms to scatter phonons
 - Novel Chalcogenides and Clathrates (Michigan State and Arizona, 1994)
 - ❖ Complex Variations of Bi_2Te_3 to reduce phonon mean-free paths
- Nano-scale Materials
 - Low-Dimensional Structures (MIT, MIT Lincoln Labs, 1992)
 - ❖ Quantum-confinement to Enhance Density of states which increase Seebeck coefficient
 - Nano-scale Superlattices (RTI, 1992)
 - ❖ **Phonon-blocking** from acoustic mismatch between superlattice components but **electron-transmitting** due to negligible electron-energy offsets
 - Heterostructure Thermionics (UCSB, Oak Ridge, 1996)
 - ❖ Thermionic-like effects using energy barriers that can be controlled in hetero-structures

New Progresses

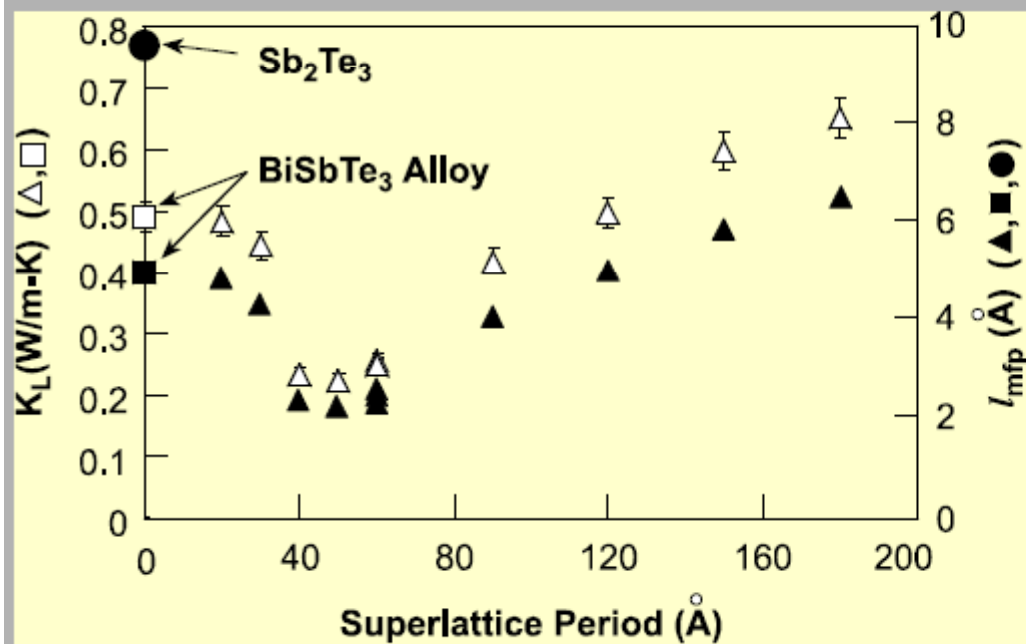
- Cs Bi₄Te₆ (Michigan State University)
 - Bulk Materials with a ZT~ 0.8 at 225K but less than 0.8 at 300K (*Science* **287**, 1024-1027, 2000)
- Filled Skuterrudites (JPL)
 - Bulk materials with a ZT ~1.35 at 900K (Proc. Of 15th International Conf. On Thermoelectrics, 1996)
- PbTe/PbTeSe Quantum-dots (Harman, MIT Lincoln Labs.)
 - ZT~ 1.6 at 300K based on cooling data (*Science* 297, Sep. 2002)
- Bi₂Te₃/Sb₂Te₃ Superlattices (RTI)
 - ZT~2.4 at 300K in devices with all properties measured at the same place, same time, with current flowing and verified by two independent techniques (*Nature*, 597-602, Oct. 2001)

Nanostructured Superlattice

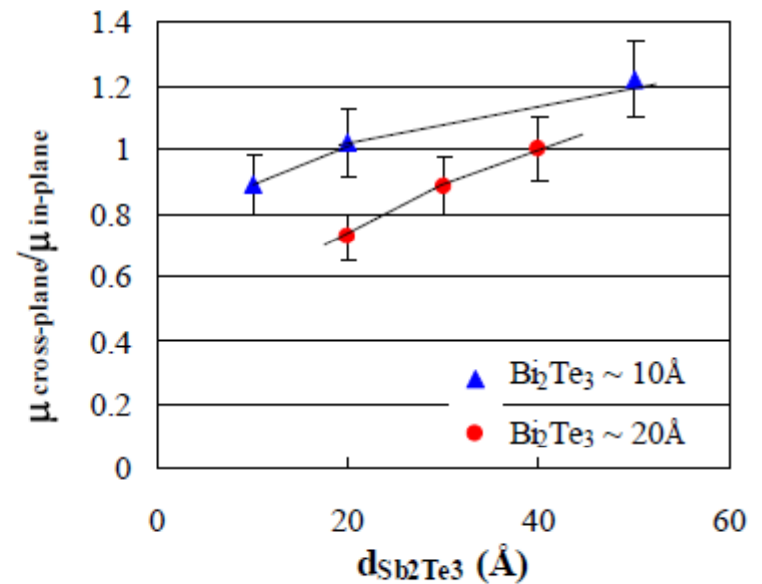


- 10Å/50Å $\text{Bi}_2\text{Te}_3/\text{Sb}_2\text{Te}_3$ Structure
- Optimized for disrupting heat transport while enhancing electron transport perpendicular to the superlattice interfaces

Lattice turning for TE

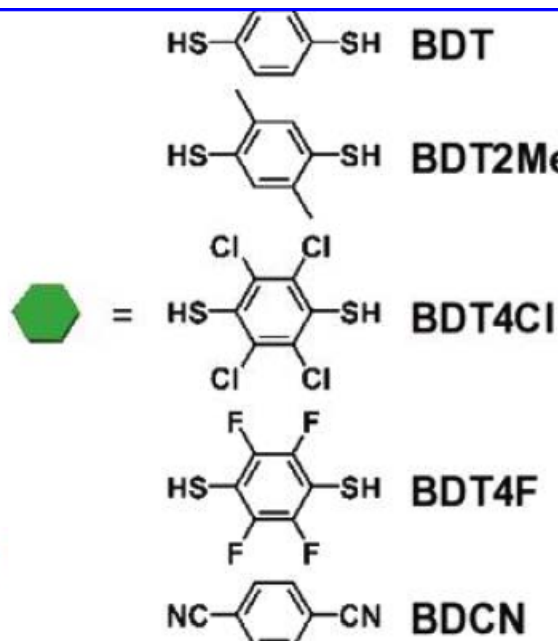
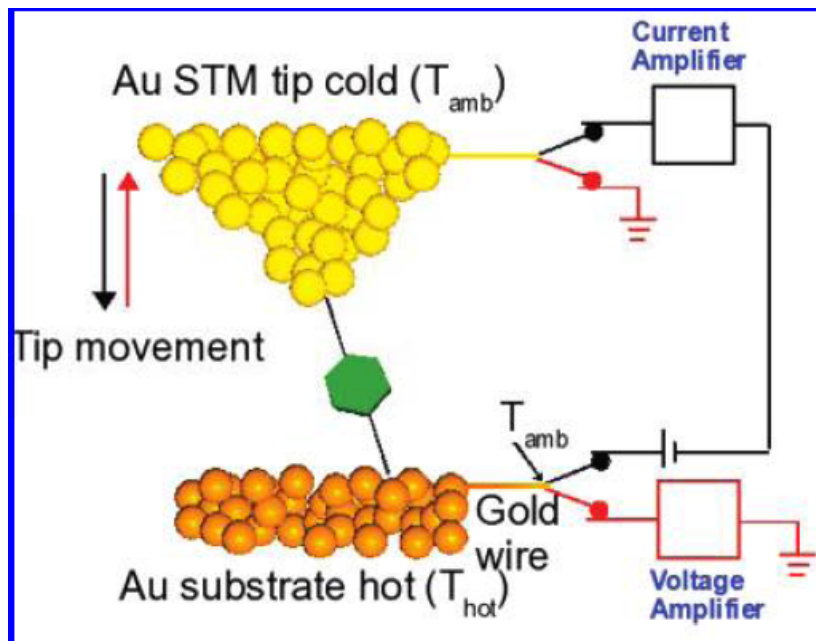


Physical Review B, 61, 3091 (2000)



Nature, 413, 597-602 (2001)

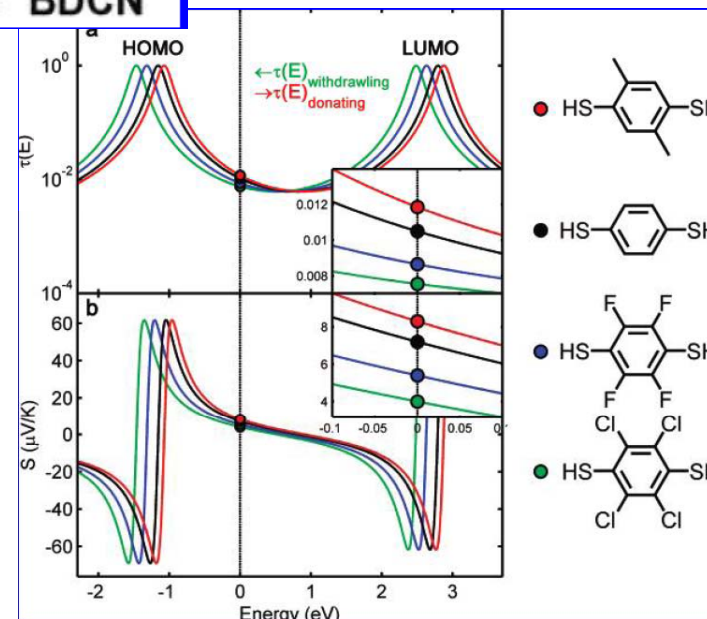
New Ideas: Metal-Organic-Metal



**NANO
LETTERS**

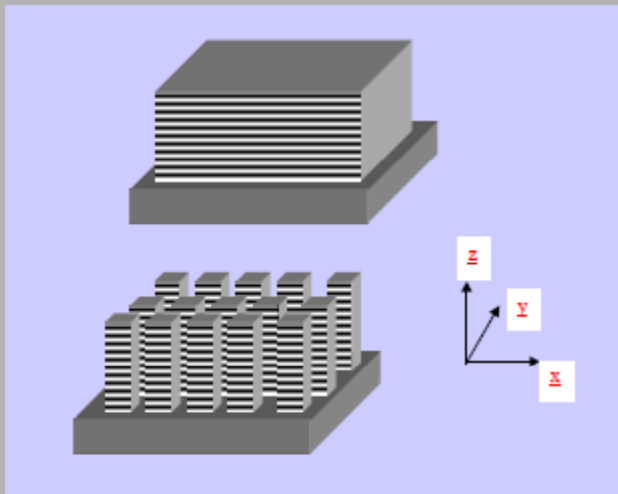
**2008
Vol. 8, No. 2
715–719**

Rachel A. Segalman

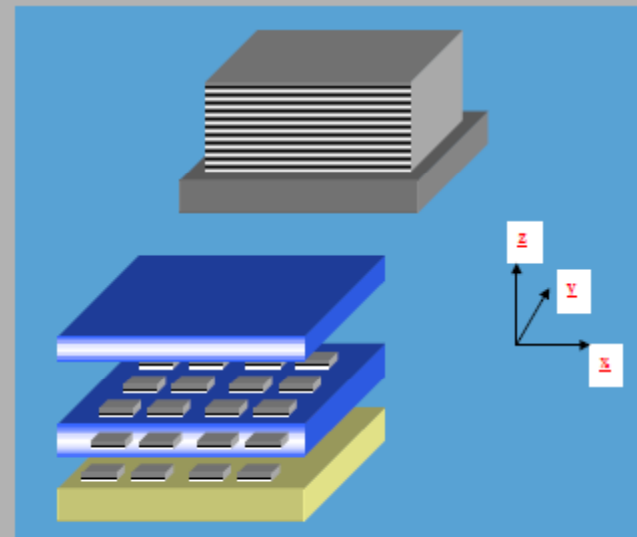


New Ideas

- Combine Phonon-Blocking, Electron-Transmitting Structures Along Heat Flow with Orthogonal Quantum-Confinement for ZT in the range of 4 to 5?

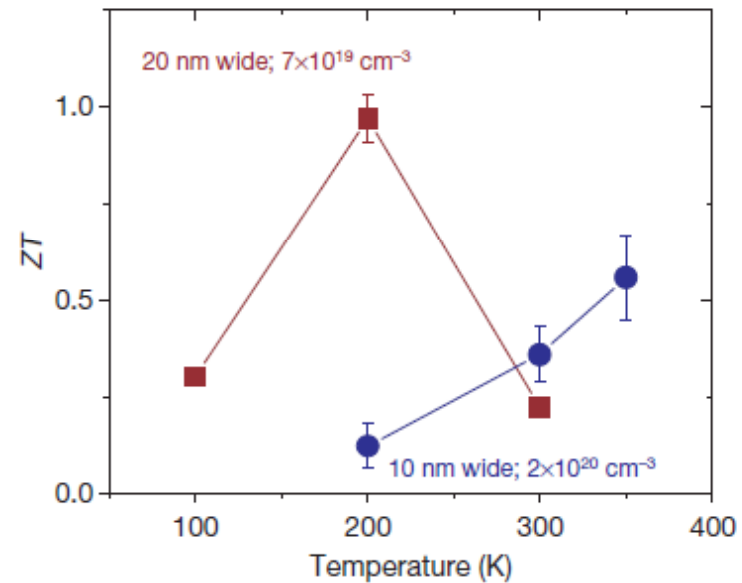
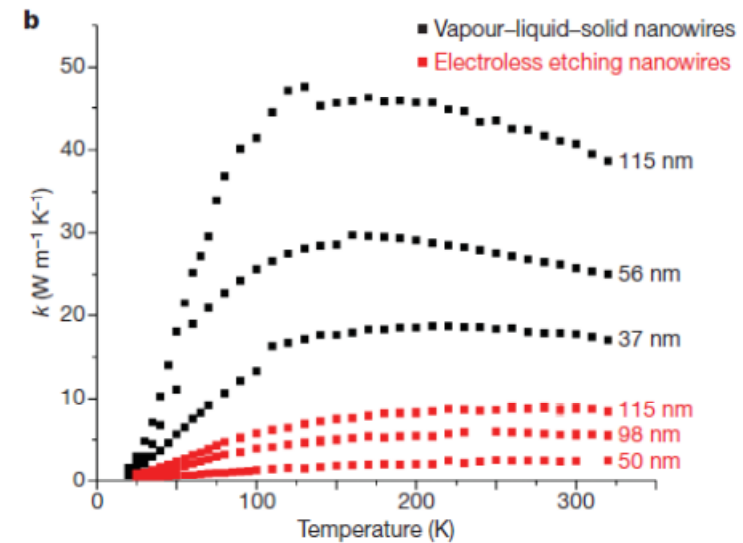
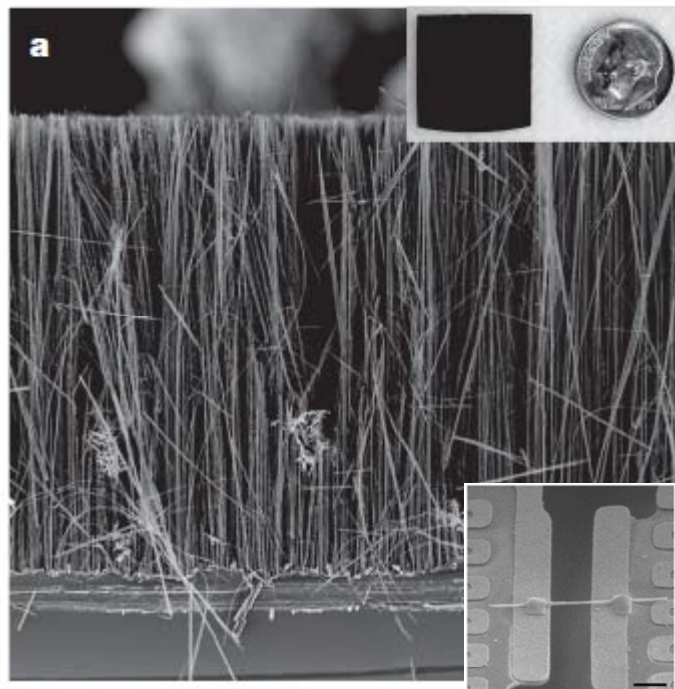
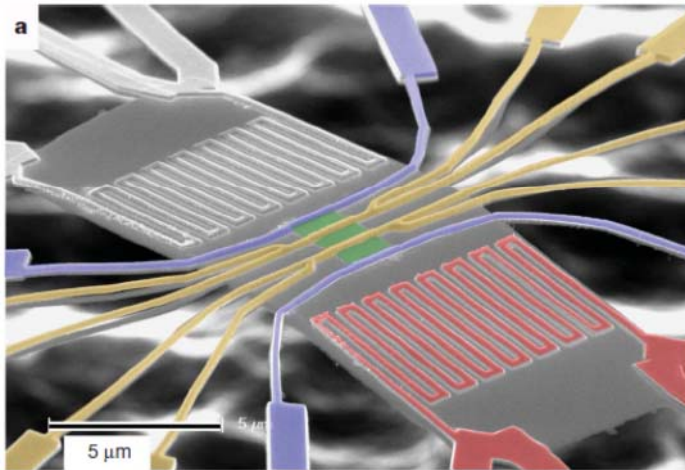


Layered SL Patterned to form 1-di Quantum-Wires – **Orthogonal Quantum Confinement**



Layered SL Patterned to form Quantum-Boxes – **Orthogonal Quantum Confinement**

Si Nanowires as an Efficient TE Material

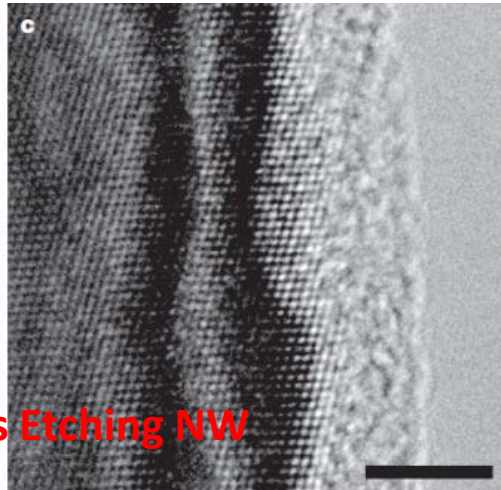


Akram I. Boukai *et al*, Nature **451** 168(2007)

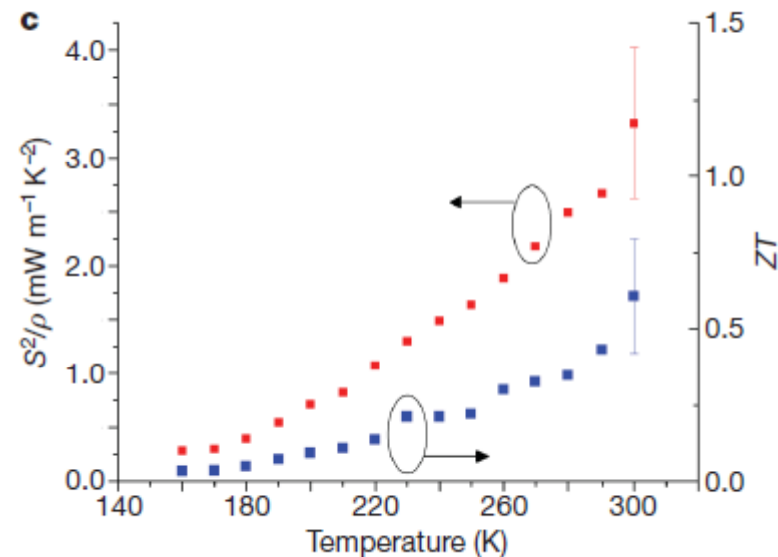
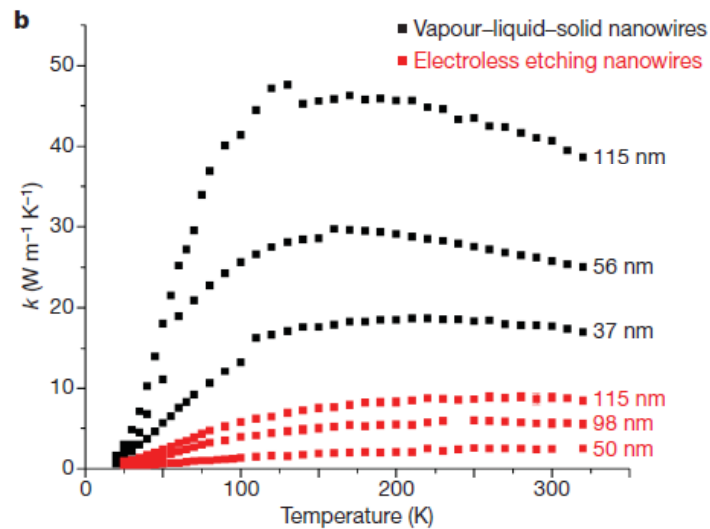
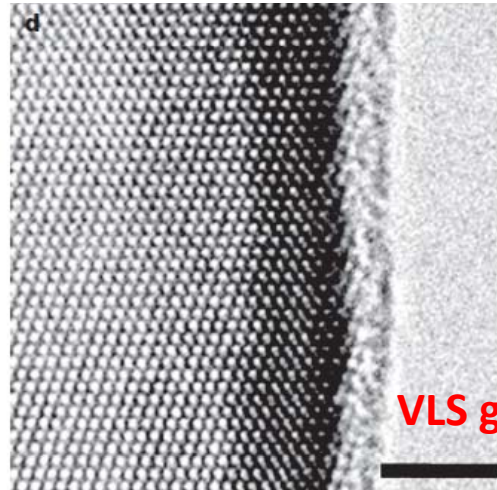
Enhanced thermoelectric performance of rough silicon nanowires

Allon I. Hochbaum^{1*}, Renkun Chen^{2*}, Raul Diaz Delgado¹, Wenjie Liang¹, Erik C. Garnett¹, Mark Najarian³, Arun Majumdar^{2,3,4} & Peidong Yang^{1,3,4}

Aqueous Etching NW



VLS grown NW



Holey Silicon as an Efficient Thermoelectric Material

Jinyao Tang,^{†,§,⊥} Hung-Ta Wang,^{†,⊥} Dong Hyun Lee,^{||} Melissa Fardy,[†] Ziyang Huo,[†] Thomas P. Russell,^{||} and Peidong Yang^{*,†,⊥,§}

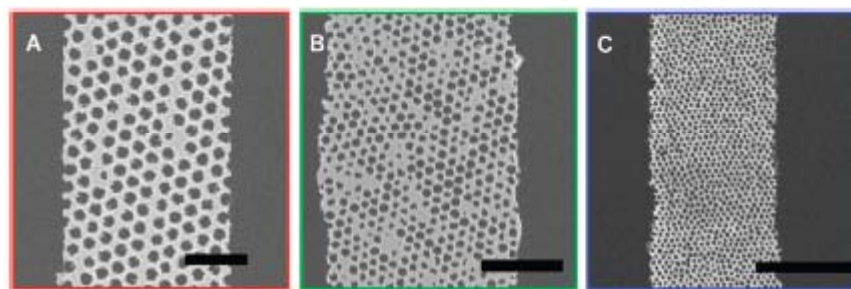
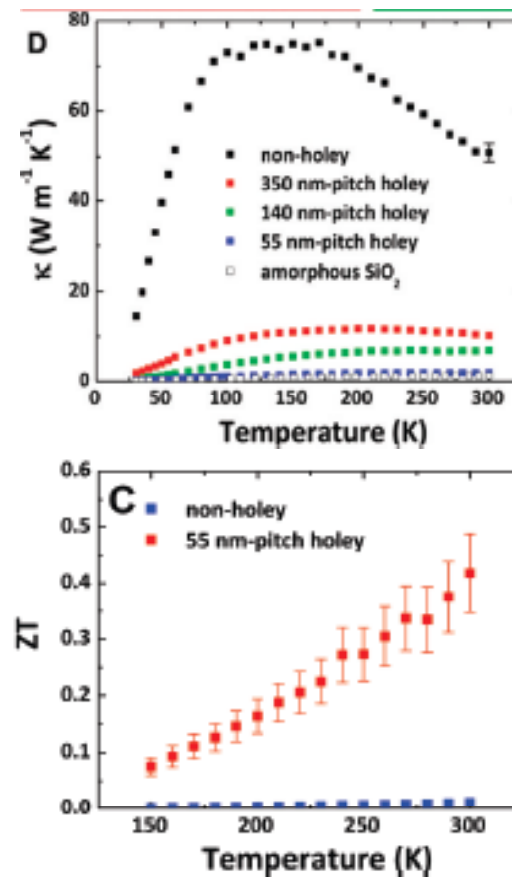
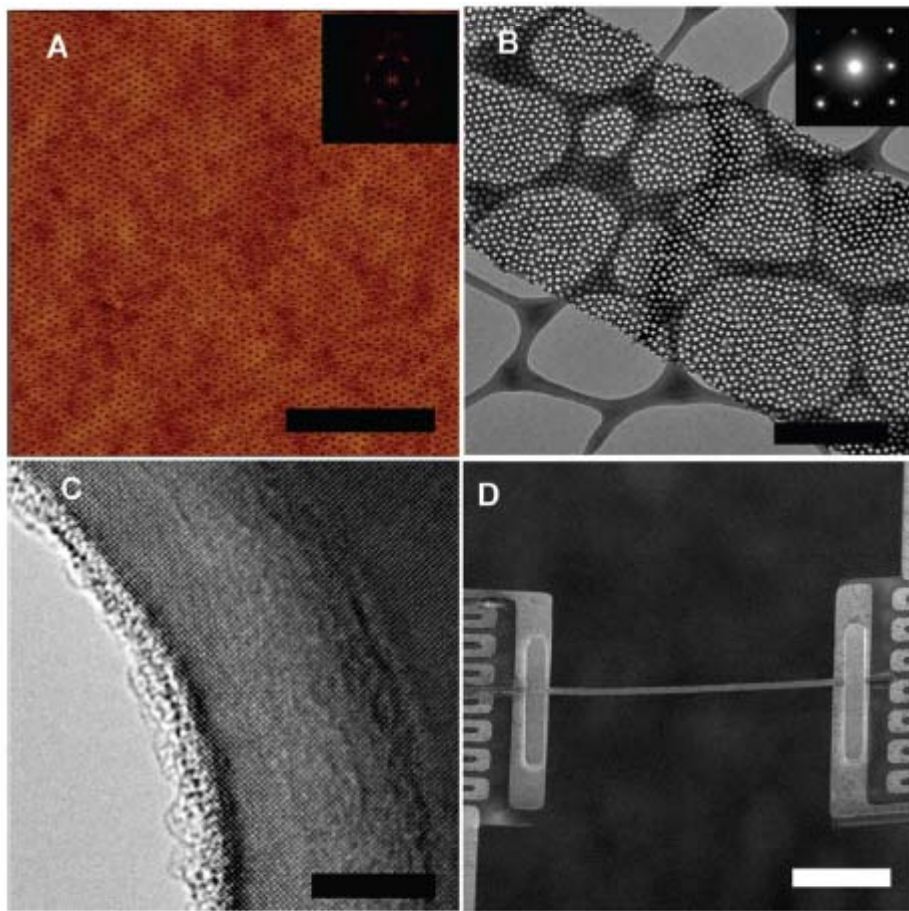
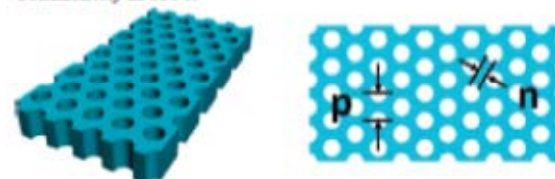
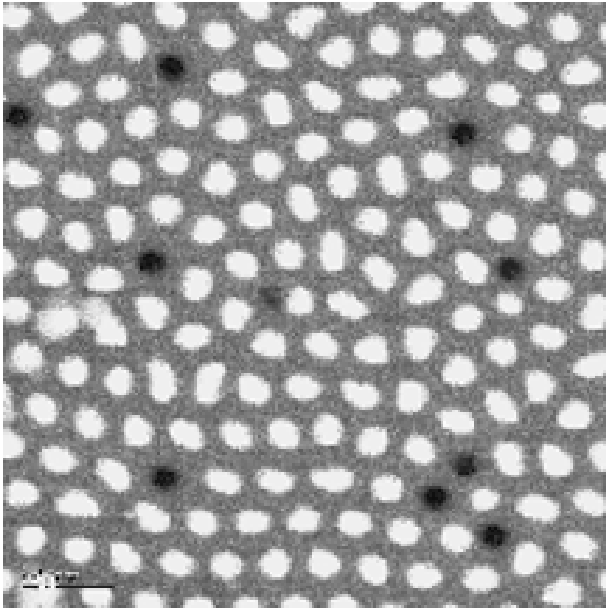


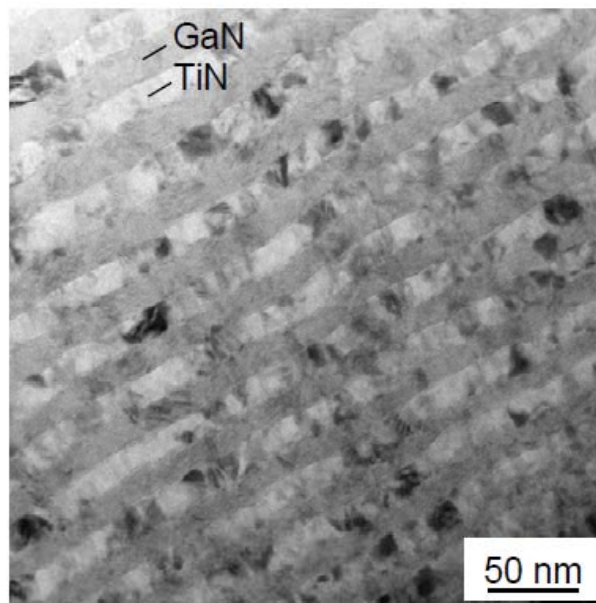
TABLE 1. Summary of the HS Geometry and the Thermal Conductivity at 300 K^a



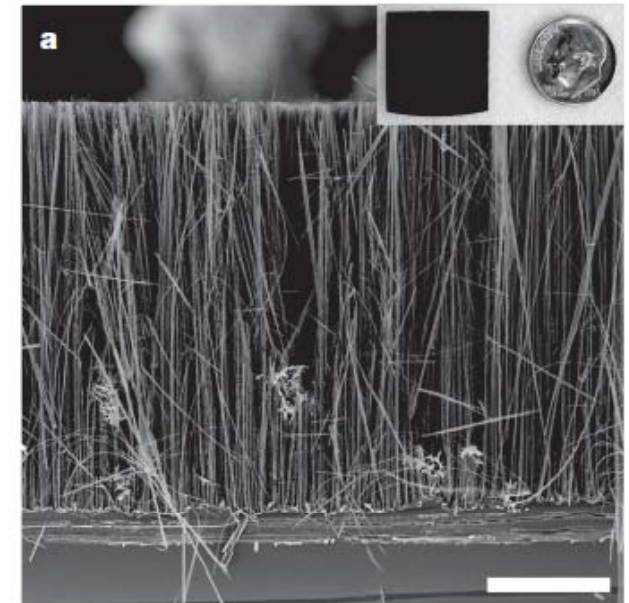
Nano-composite & Nanowires



Nano-composite



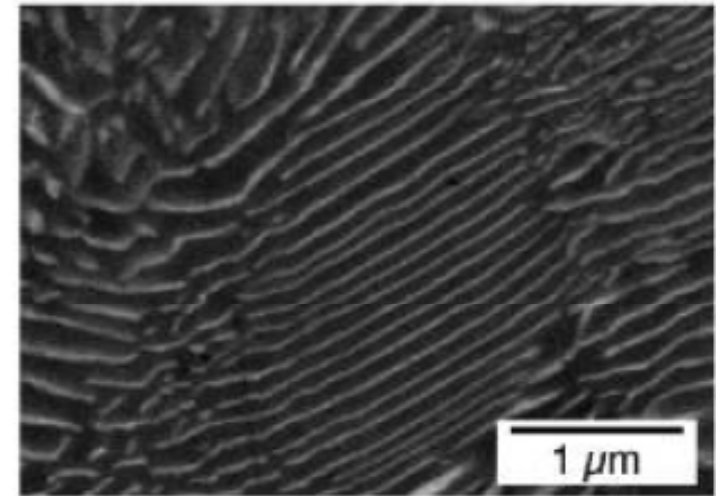
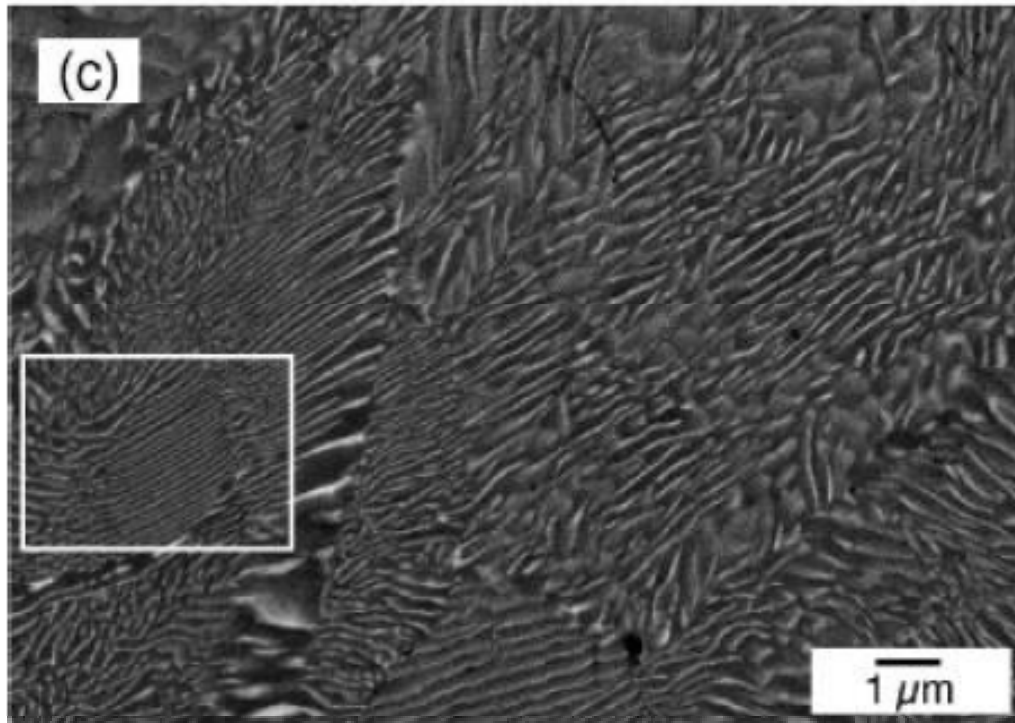
Super-lattice structure



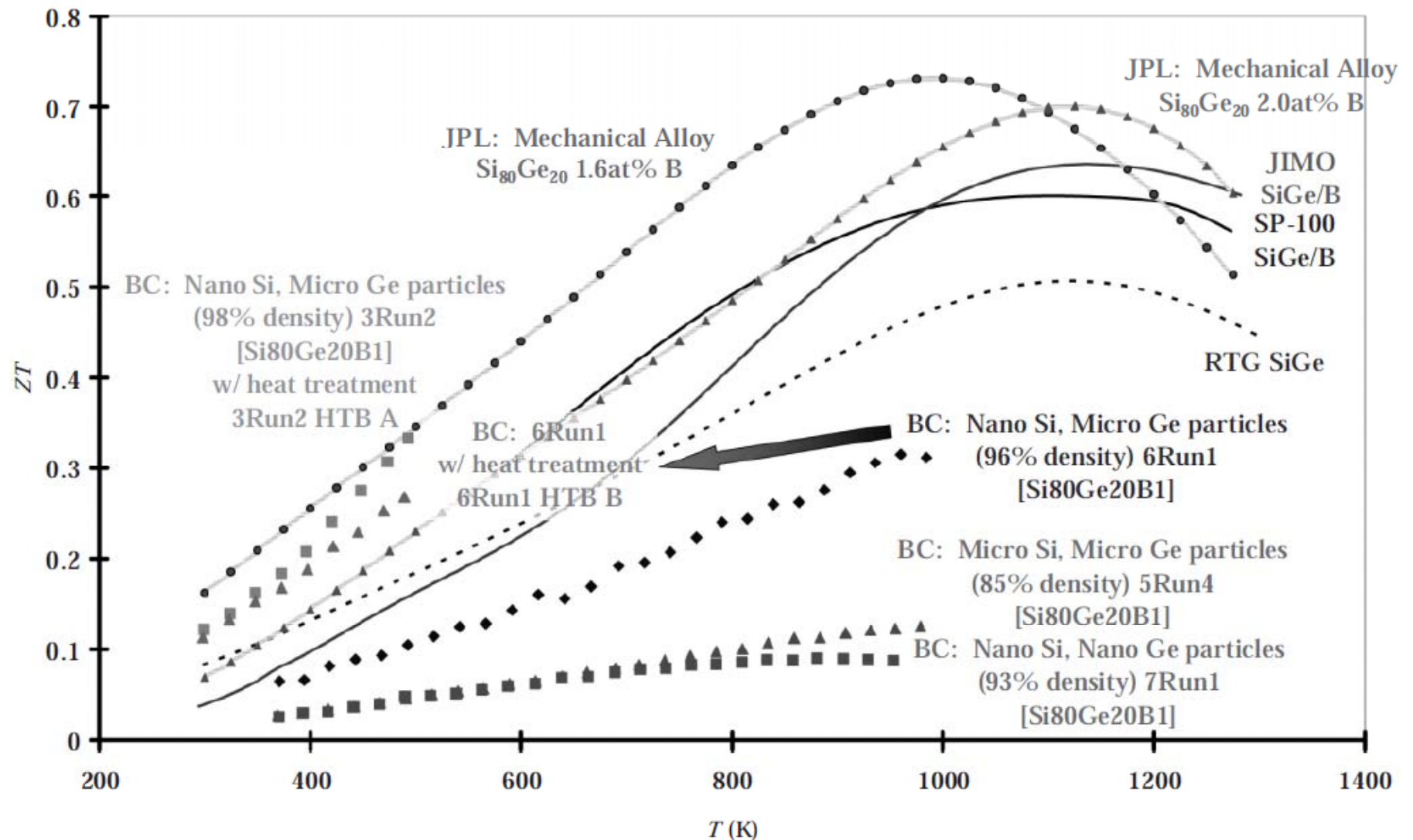
Nanowires

Goal: Reduce Thermal conductivity or
Increase electrical conductivity

Lamellae of Thermoelectric PbTe and Sb₂Te₃



ZT of $\text{Si}_{0.8}\text{Ge}_{0.2}\text{B}_{0.016}$ by ball milling



1-D Nano Materials in AML

JACS 123, 2791 (2001)

APL 81, 22 (2002)

JACS 127, 2820 (2005)

APL 90, 213104 (2007)

Adv. Func. Mater. (2008)

Small 4, 925 (2008)

Electrochem. Comm. 11, 850 (2009)

Anal. Chem. 81, 36 (2009)

Angew. Chem. Int. Ed. 49, 5966 (2010)

APL 81, 4189 (2002)

Adv. Func. Mater. 12, 687 (2002)

APL 86, 203119 (2005)

JACS 128, 8368 (2006)

PRB 75, 195429 (2007)

J. Power Sources 171, 55 (2007)

JACS 130, 3543 (2008)

J. Power Sources 190, 279 (2009)

Chapter 9, pp. 259-309, Nanowires

and nanobelts, Kluwer (2004)

Adv. Func. Mater. 16, 537 (2006)

APL 90, 123109 (2007)

Adv. Mater. 19, 4524 (2007)

Opt. Express 17, 11690 (2009)



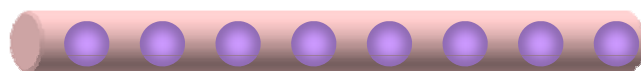
Wire/Rod



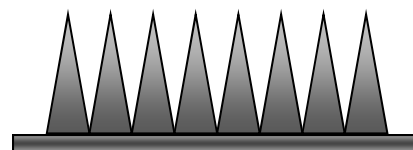
Tube



Belt



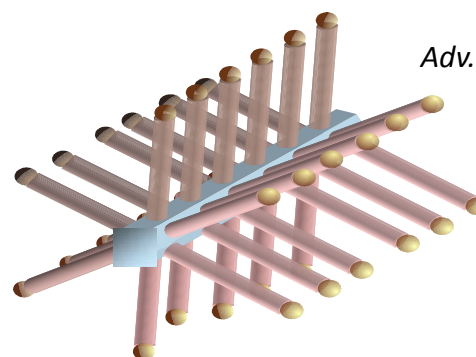
Peapod



Nanotip



Core-shell



Brush

APL 83, 1420 (2003)

Nano. Lett. 4, 471 (2004)

Chem. Mater. 17, 553 (2005)

Adv. Func. Mater. 15, 783 (2005)

APL 86, 203119 (2005)

US Patent 6,960,528,B2

APL 89, 143105 (2006)

Nature Nanotech. 2, 170 (2007)

Nano Lett. 9, 1938 (2009)

APL 81, 1312 (2002)

Nano. Lett. 3, 537 (2003)

Adv. Func. Mater. 14, 233 (2004)

Other related:

APL 86, 21911 (2005)

APL 86, 83104 (2005)

APL 86, 161901 (2005)

APL 87, 261915 (2005)

JVST B 24, 87 (2006)

APL 88, 73515 (2006)

Adv. Mater. 21, 259 (2009)

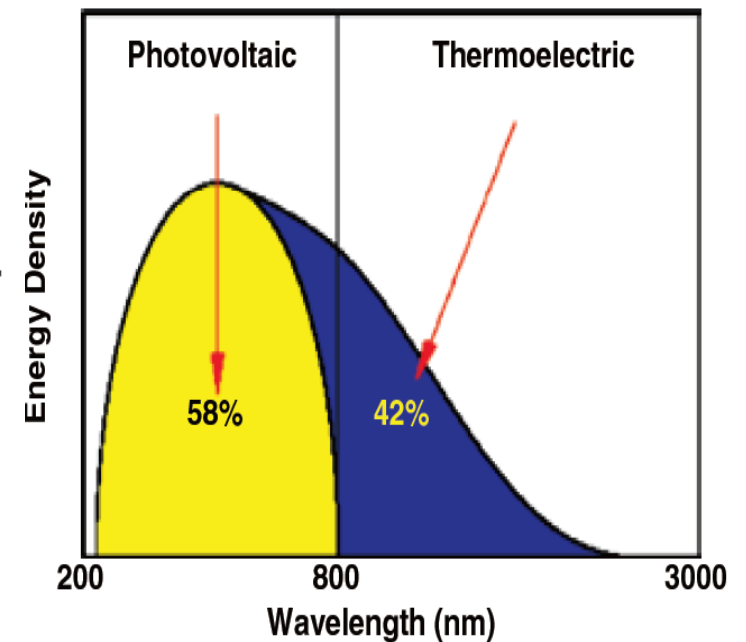
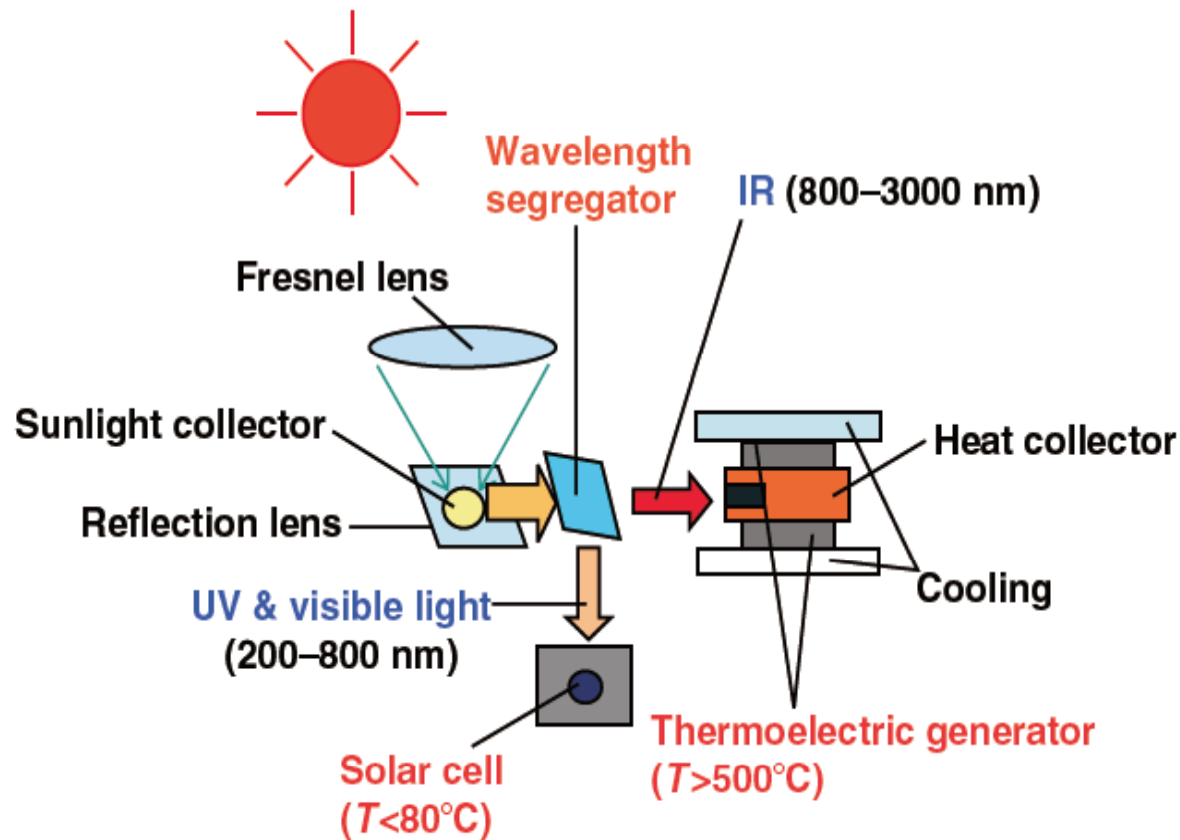
Angew. Chem. 48,7586 (2009)

J. Mater. Chem. (2010)

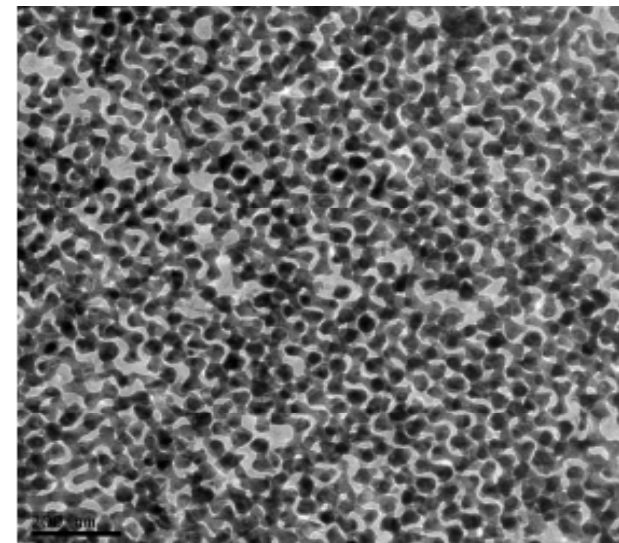
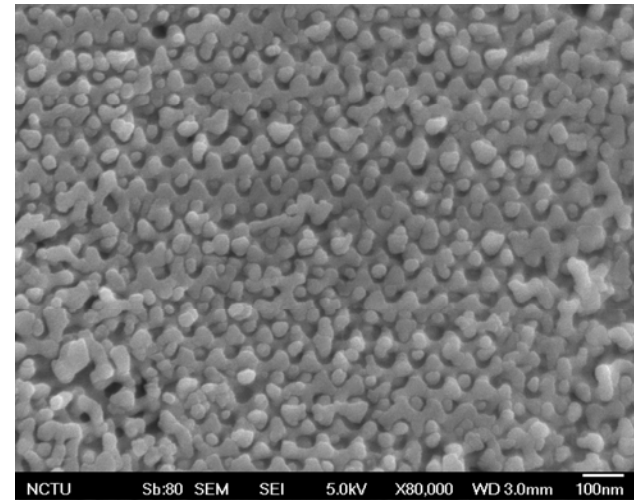
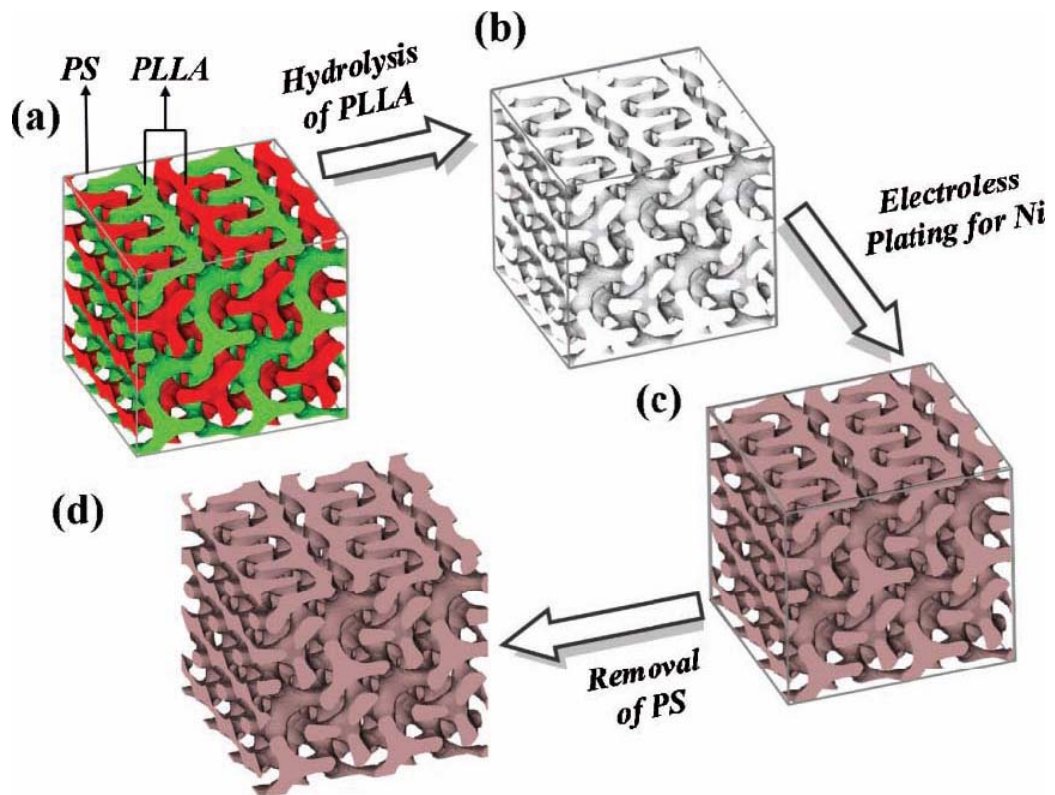
Adv. Mater. 14, 1847 (2002)

Nature Mater. 5, 102 (2006)

Applications: Solar Harvesting



Nanocomposites



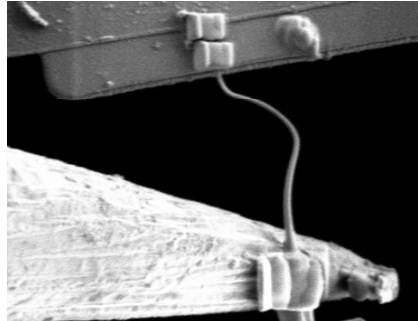
TE Measurement



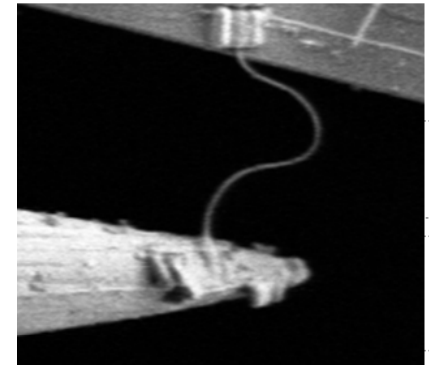
Multiwall carbon
nanotube



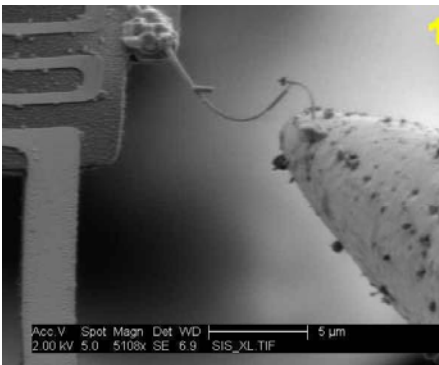
Silver nanowire



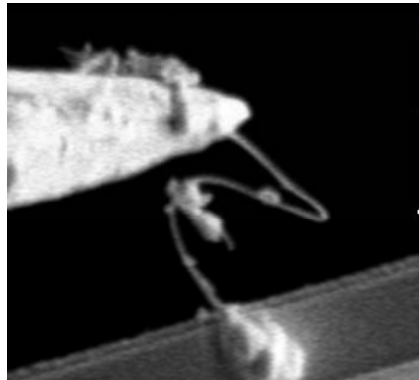
ZnO nanowire



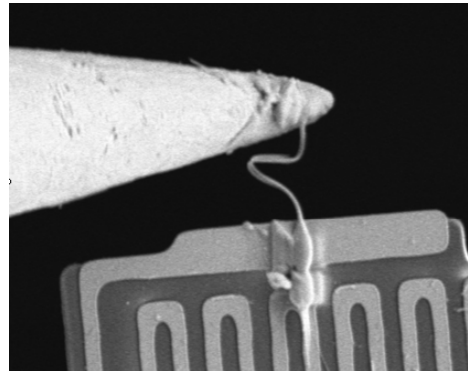
Ga₂O₃
nanowire



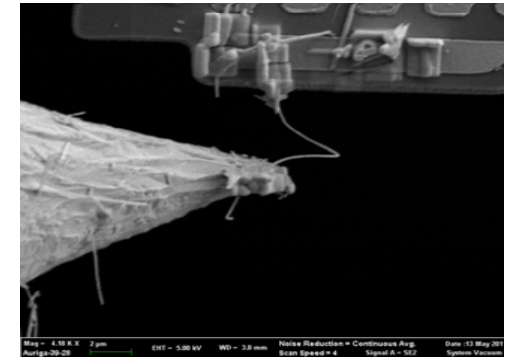
Undoped Si
nanowire



P-type Si
nanowire



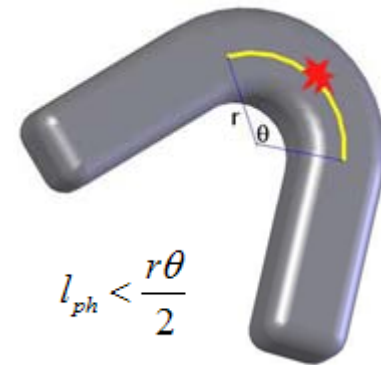
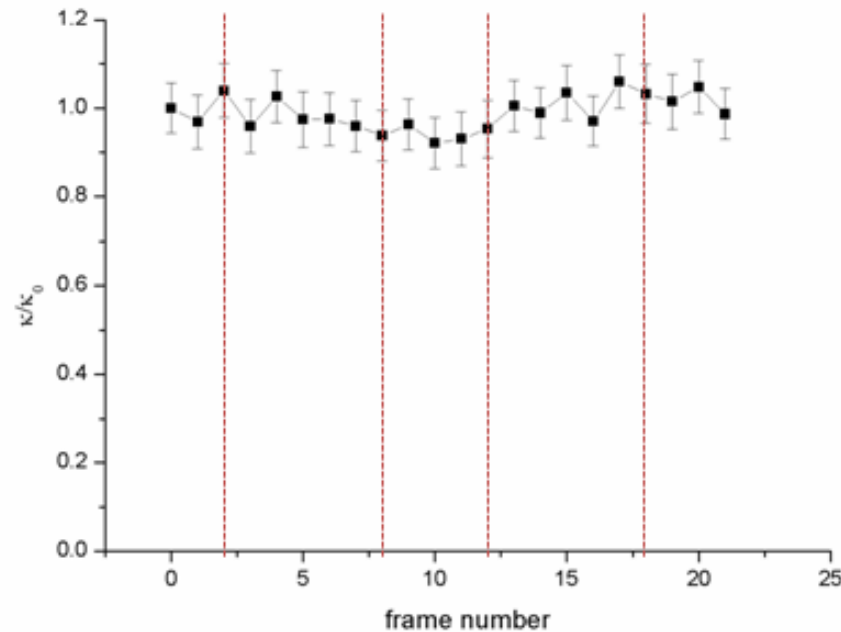
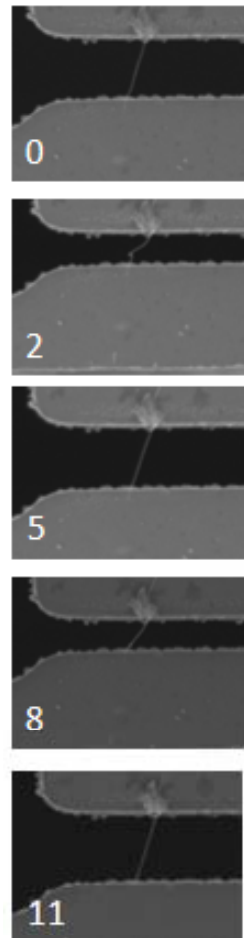
N-type Si
nanowire



SiGe nanowire

Thermal conductivities remain intact under extreme strain

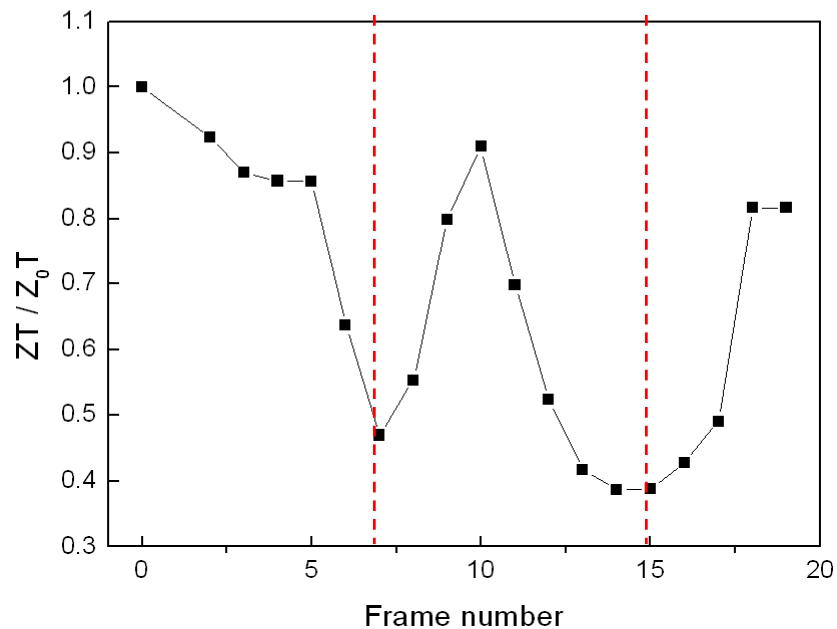
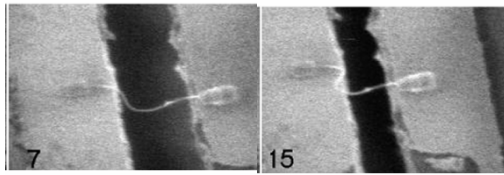
SWCNT goes beyond the theoretical limit of 1D ballistic phonon transport



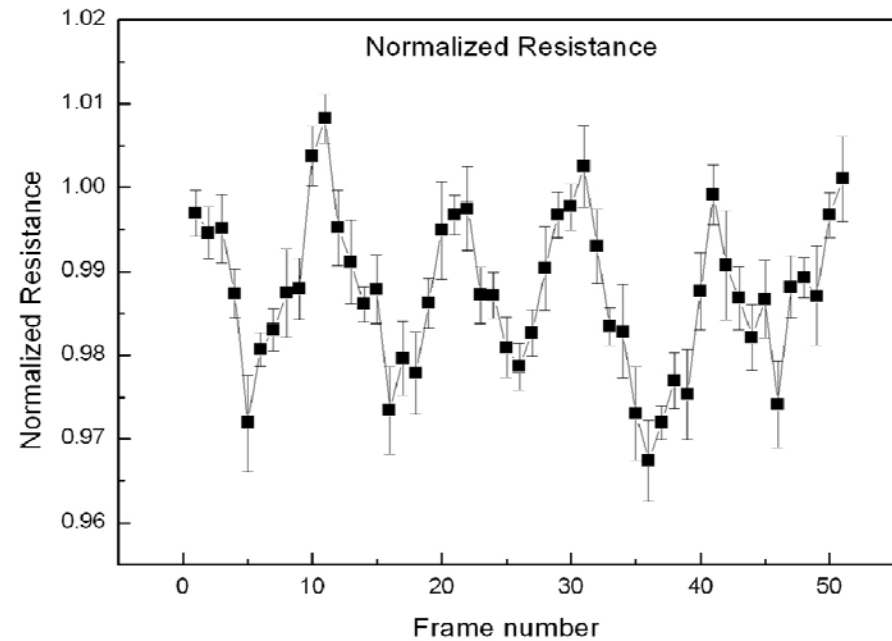
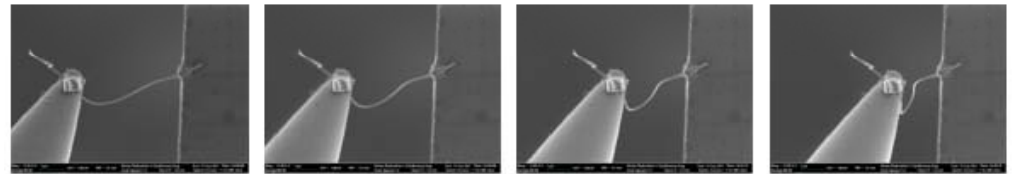
No changes of thermal conductivity is observed even when the bending radius of curvature is less than the phonon mean free path.

Modulation of ZT by Strain

(Possible active TE device!)



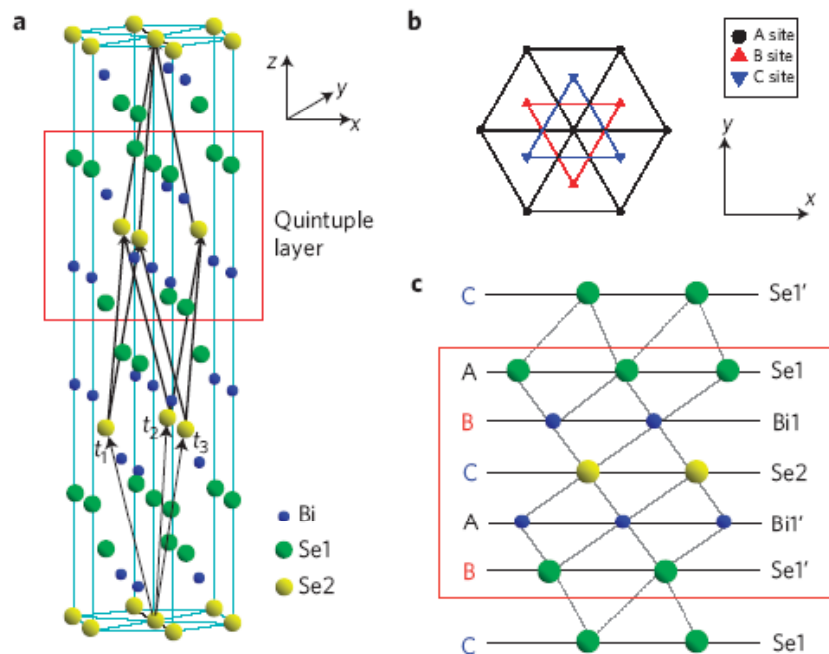
Carbon nanotube



Silicon nanowire (p-type)

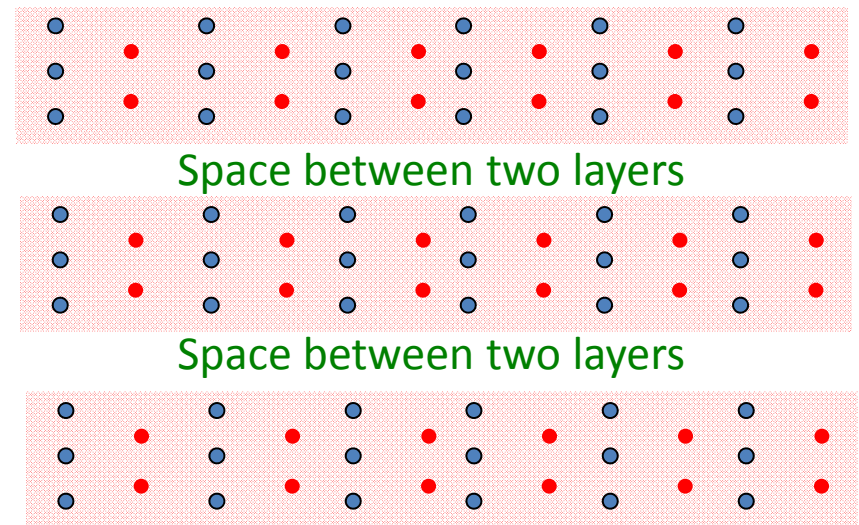
Thermopower responses are currently under investigation.

Crystal Structure of Bi_2Se_3



Crystal structure of Bi_2Se_3

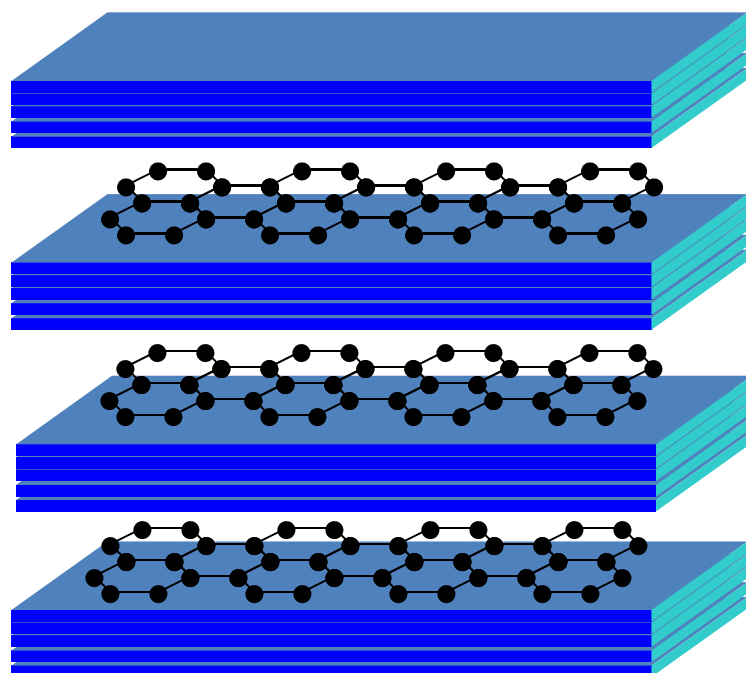
Bi_2Se_3 cleaves easily along the trigonal axis due to van der Waals bonding between neighboring selenium layers.



Haijun Zhang *et al*, Nature materials **5** 438 (2009)

F. C. Chou's lab

Graphene-Intercalation in Bi_2Se_3

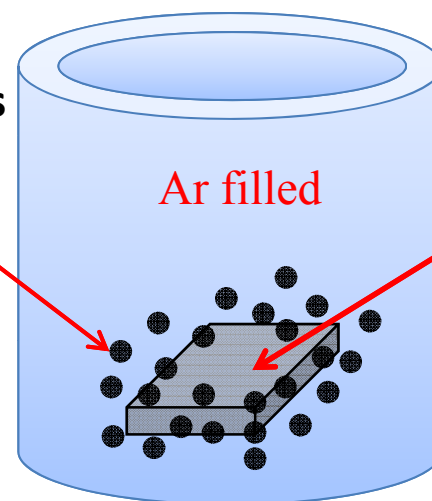


Bi_2Se_3



One or few layer graphite

Amorphous carbon



Bi_2Se_3
Single Crystal

Tantalum crucible

Annealed under ~ 2 atm, 400°C , 2 weeks.

Bi_2Se_3 crystal well packed with carbon

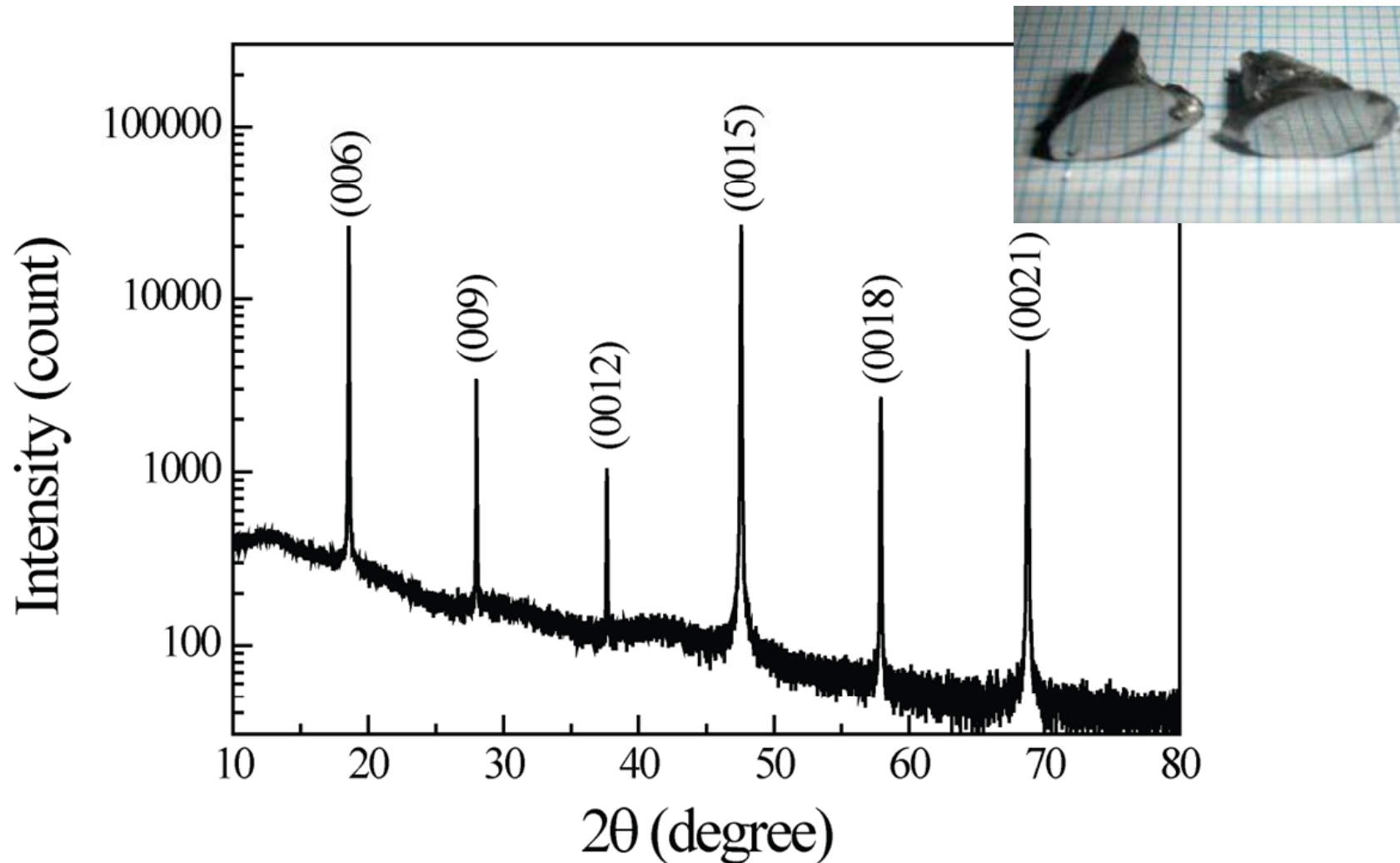
Bi_2Se_3

Soft

G-intercalated Bi_2Se_3

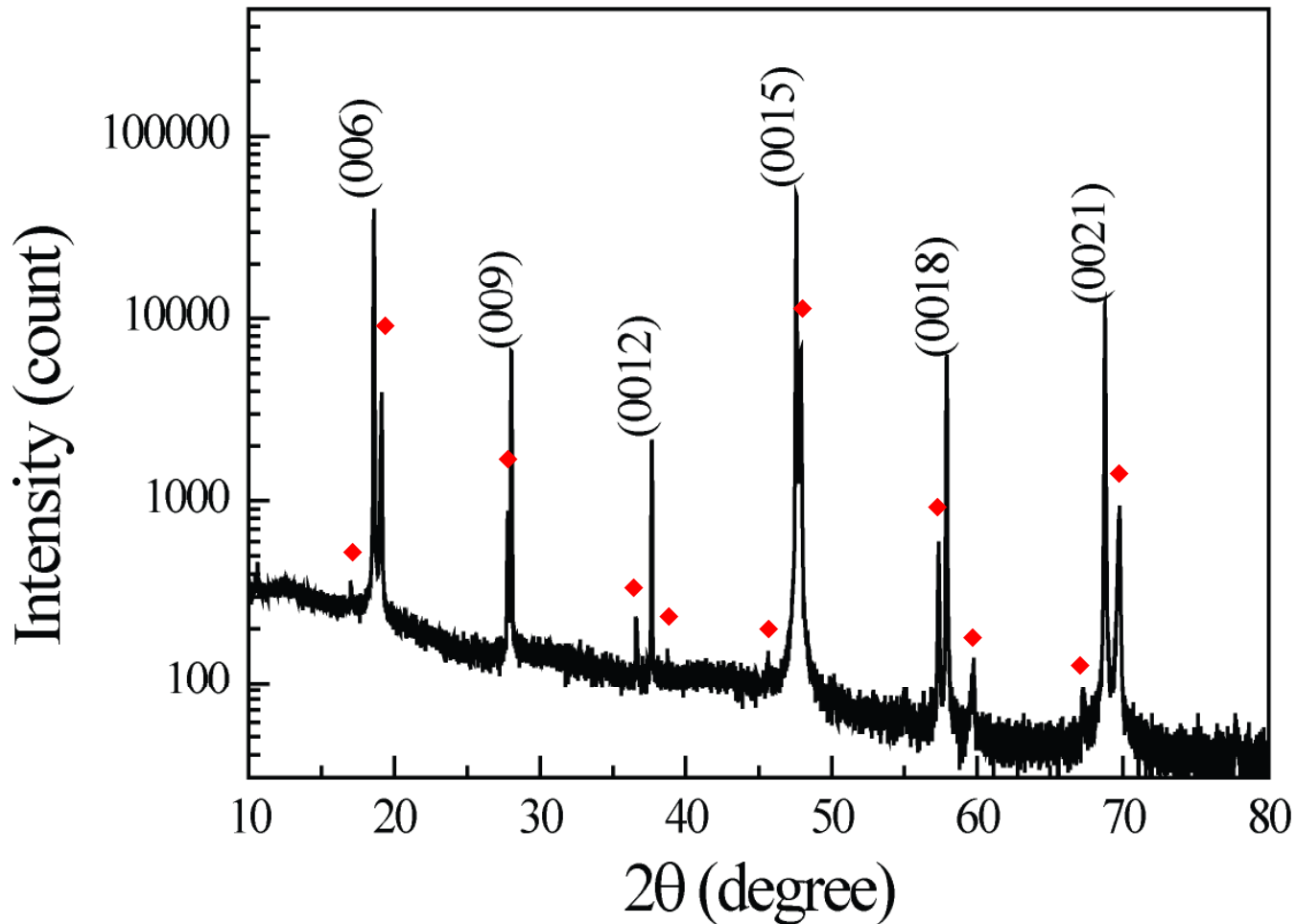
Brittle

XRD Pattern of Bi_2Se_3 Single Crystal

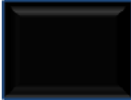


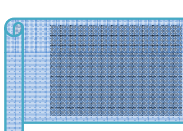
XRD pattern of pure Bi_2Se_3 depicts its superior crystal quality.

XRD Pattern of Bi_2Se_3 After Intercalation



XRD pattern of Bi_2Se_3 with carbon intercalated.
Unknown peaks near main peaks can be observed.

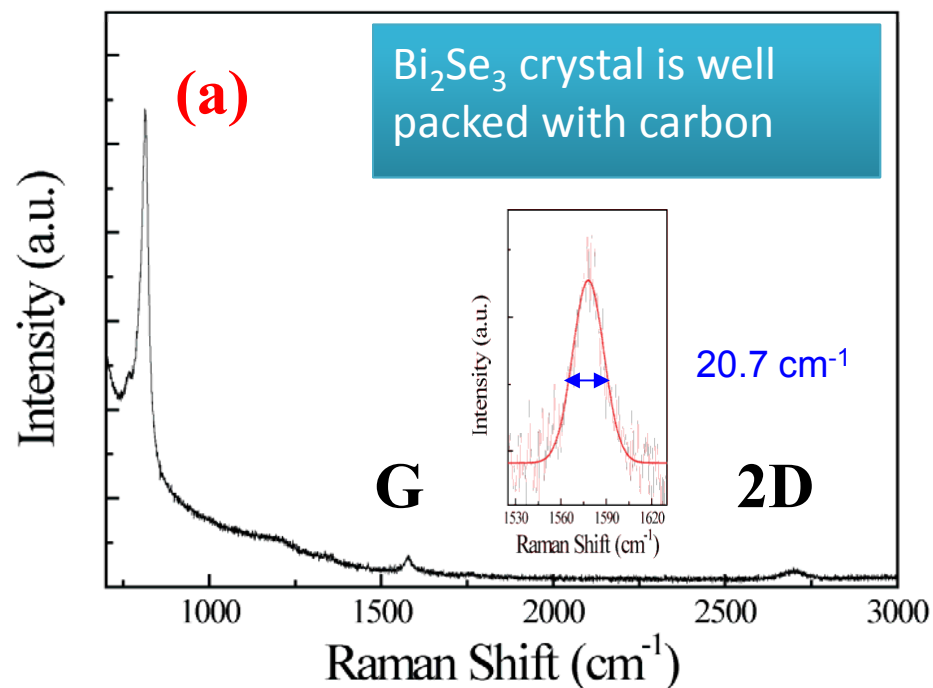
 Bi_2Se_3
with C

 Tape

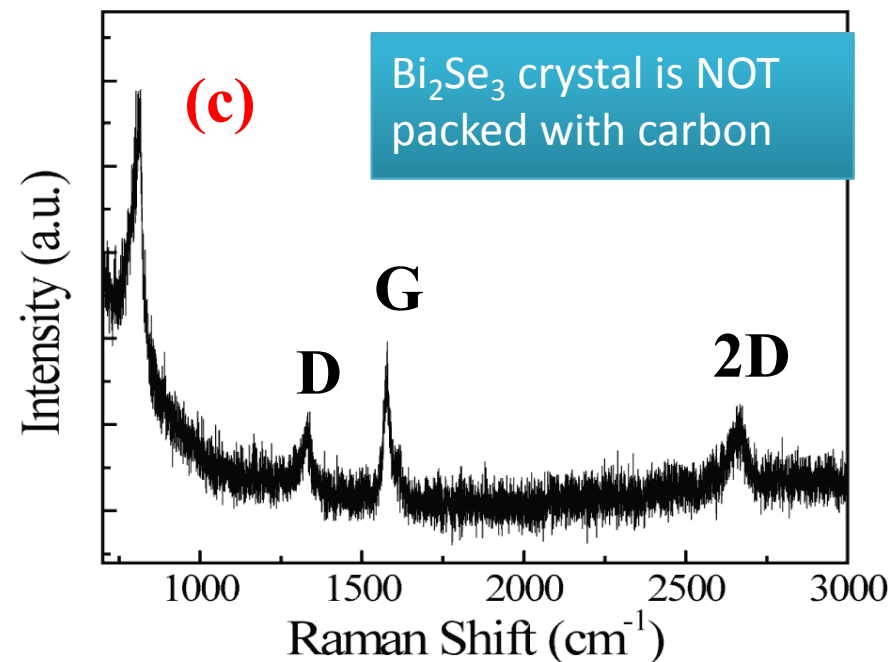
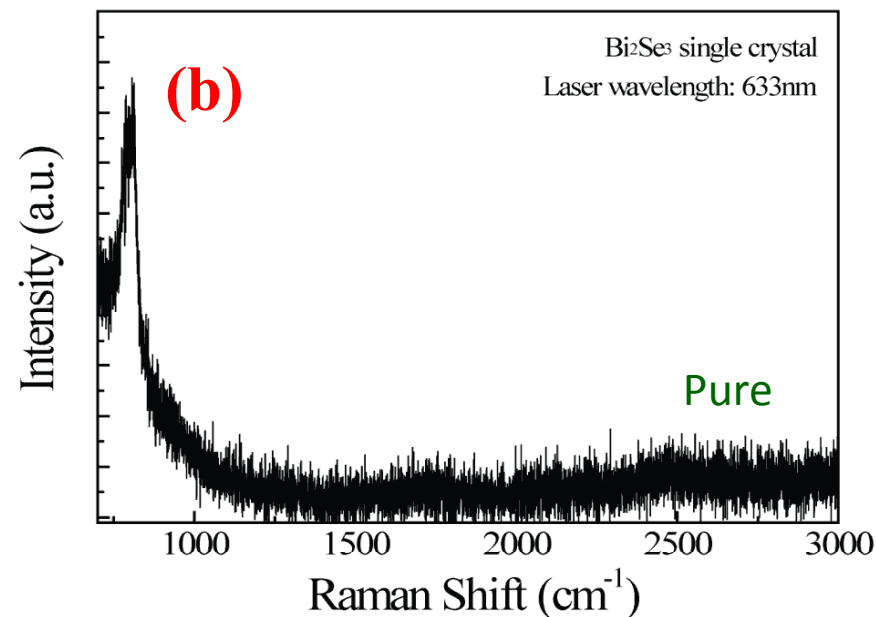


The Bi_2Se_3 layer can be
flaked off from the crystal.

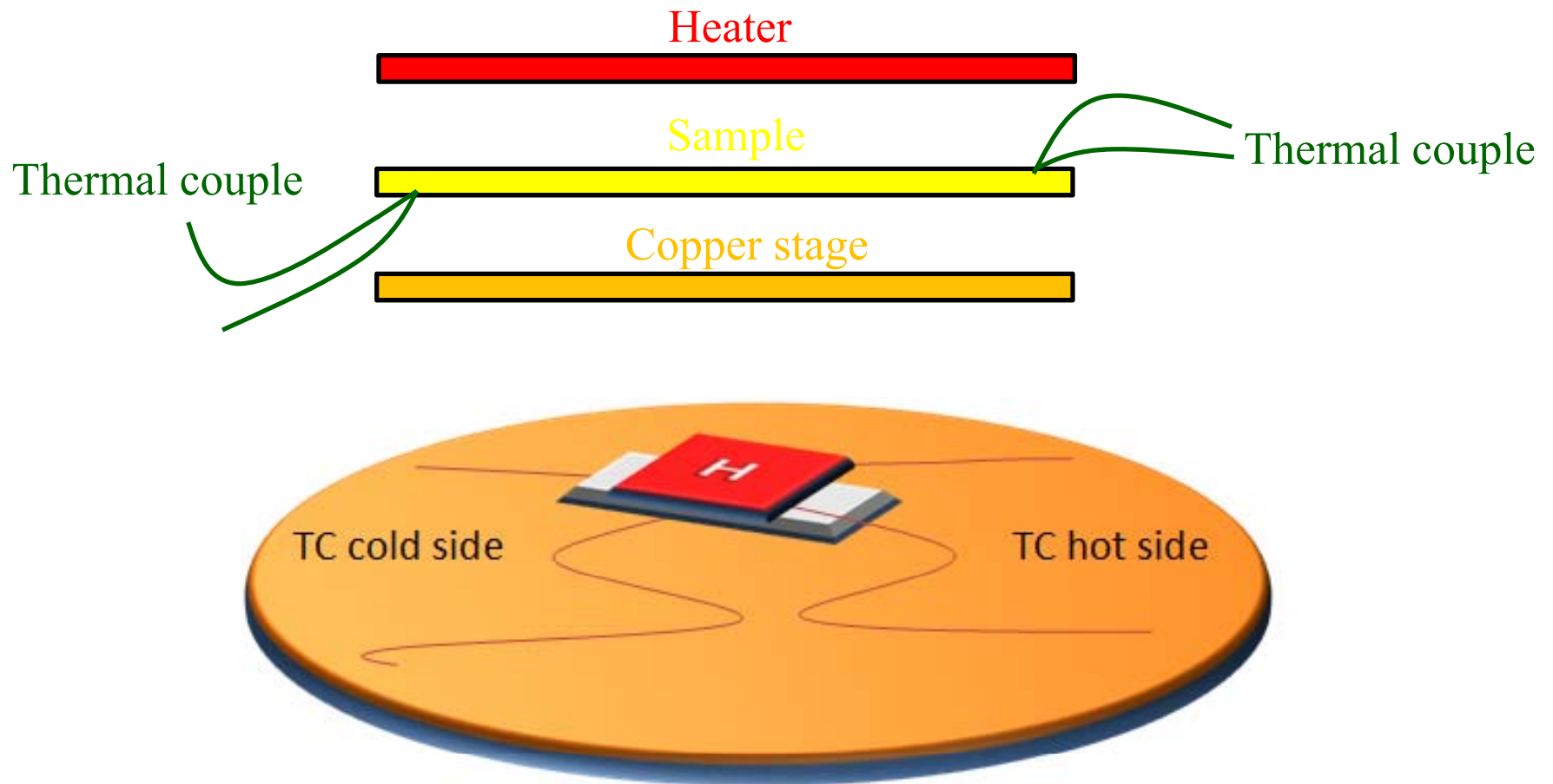
Raman Spectrum of Bi_2Se_3 with G-Intercalation



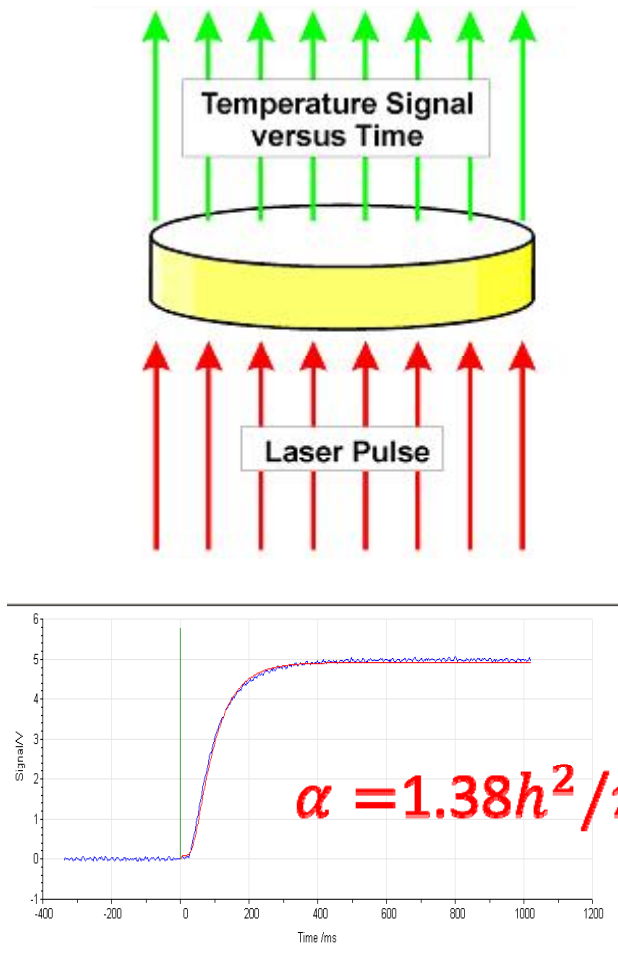
The Raman spectrum of carbon-intercalated Bi_2Se_3 . Amorphous carbon diffused into crystal and formed graphite-like structure.



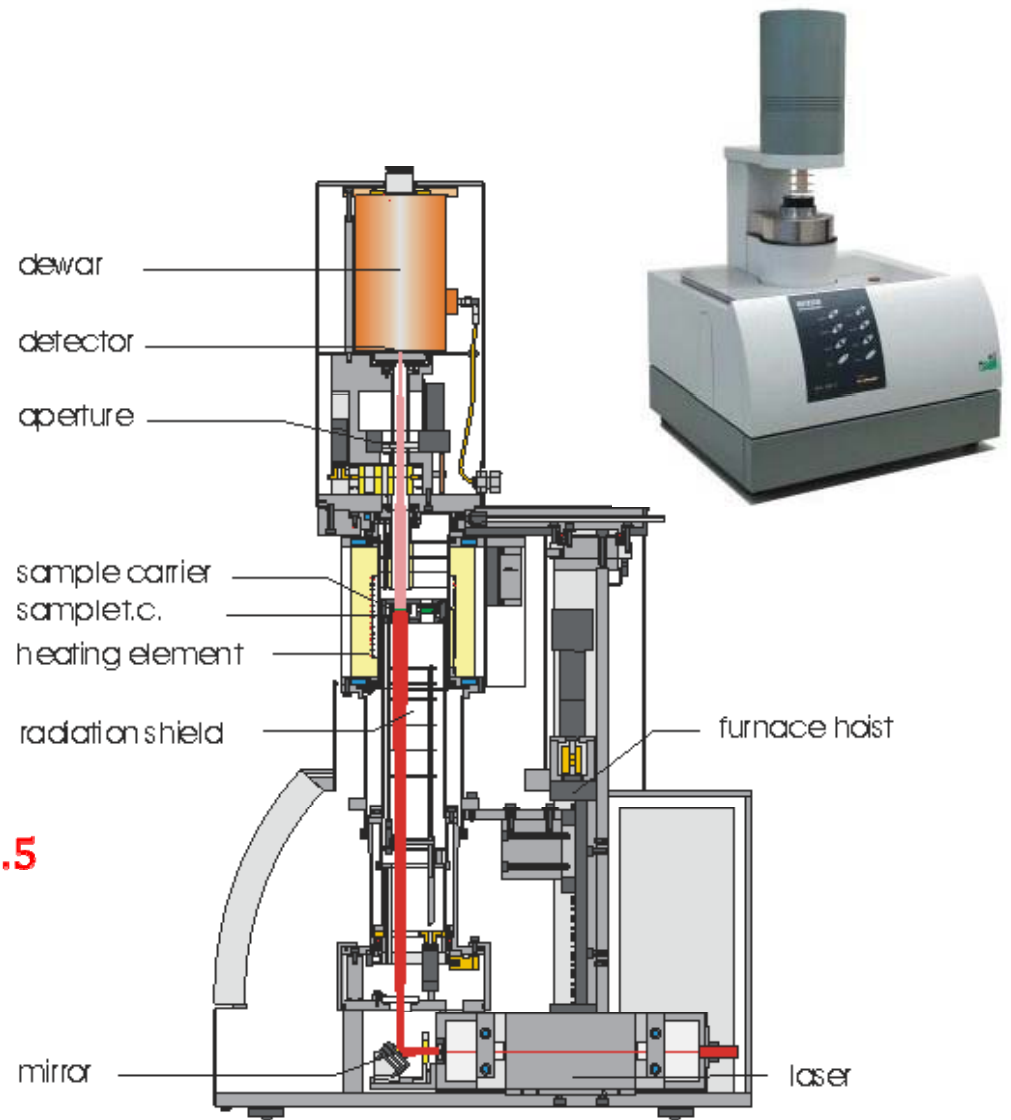
TE Technique: Seebeck Coefficient & Electrical Conductivity



Thermal Diffusivity Measurement

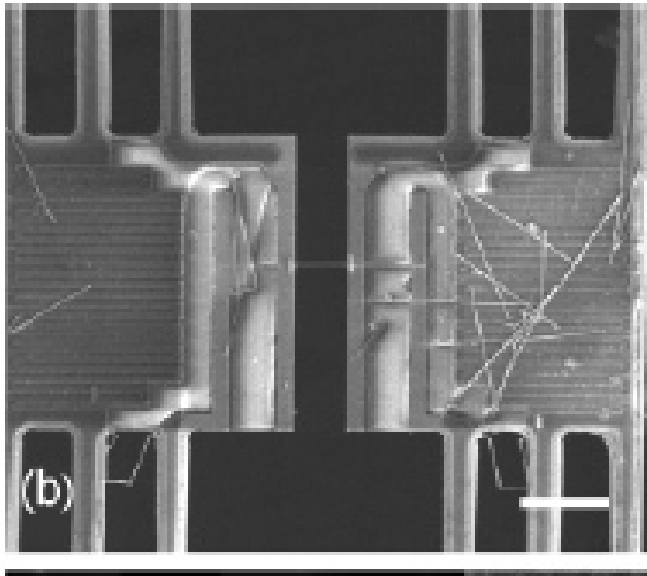


We can get the thermal diffusivity by measure the “half-time” of temperature increase resulted from laser heating.



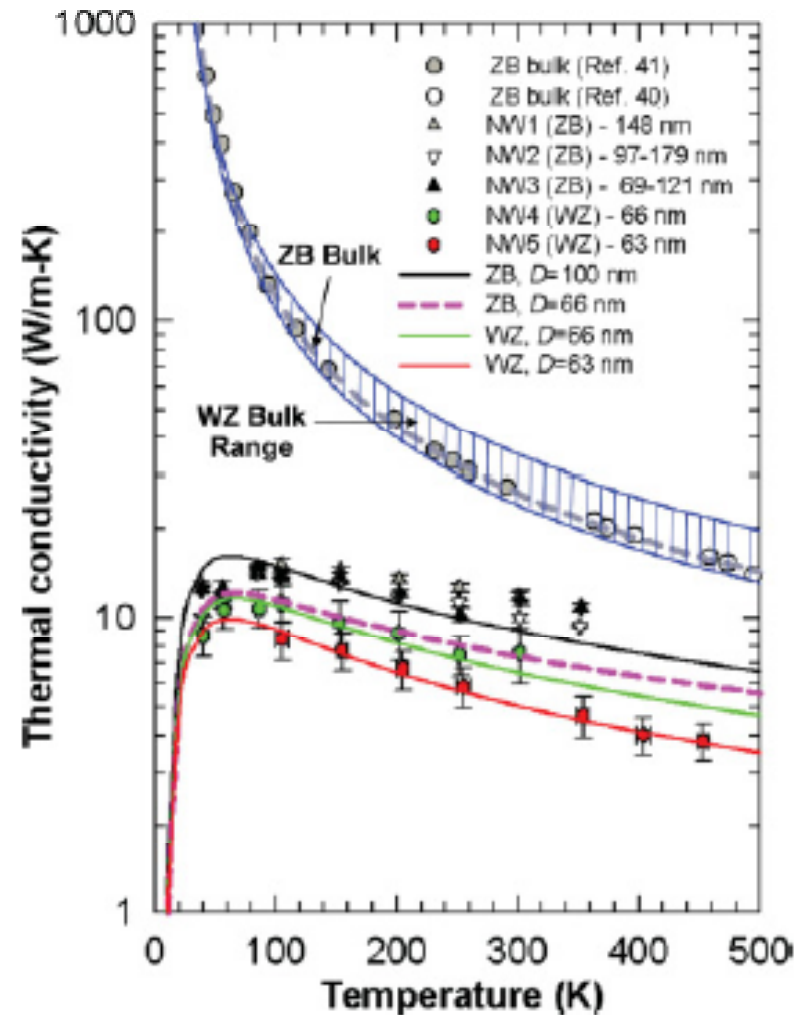
Cooperator: Dr. Y. Chen's lab

Reduction of Thermal Conductivity in Nanowires



The Seebeck coefficient, electrical conductivity, and thermal conductivity, can be measured via using the micro-thermometer device.

(Cooperator: Chih-Wei Chang)



Thermal conductivities of InAs nanowires with different diameters.

Power Factor Enhancement of Nanowires

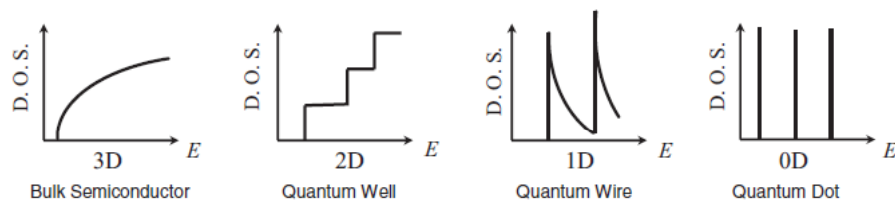
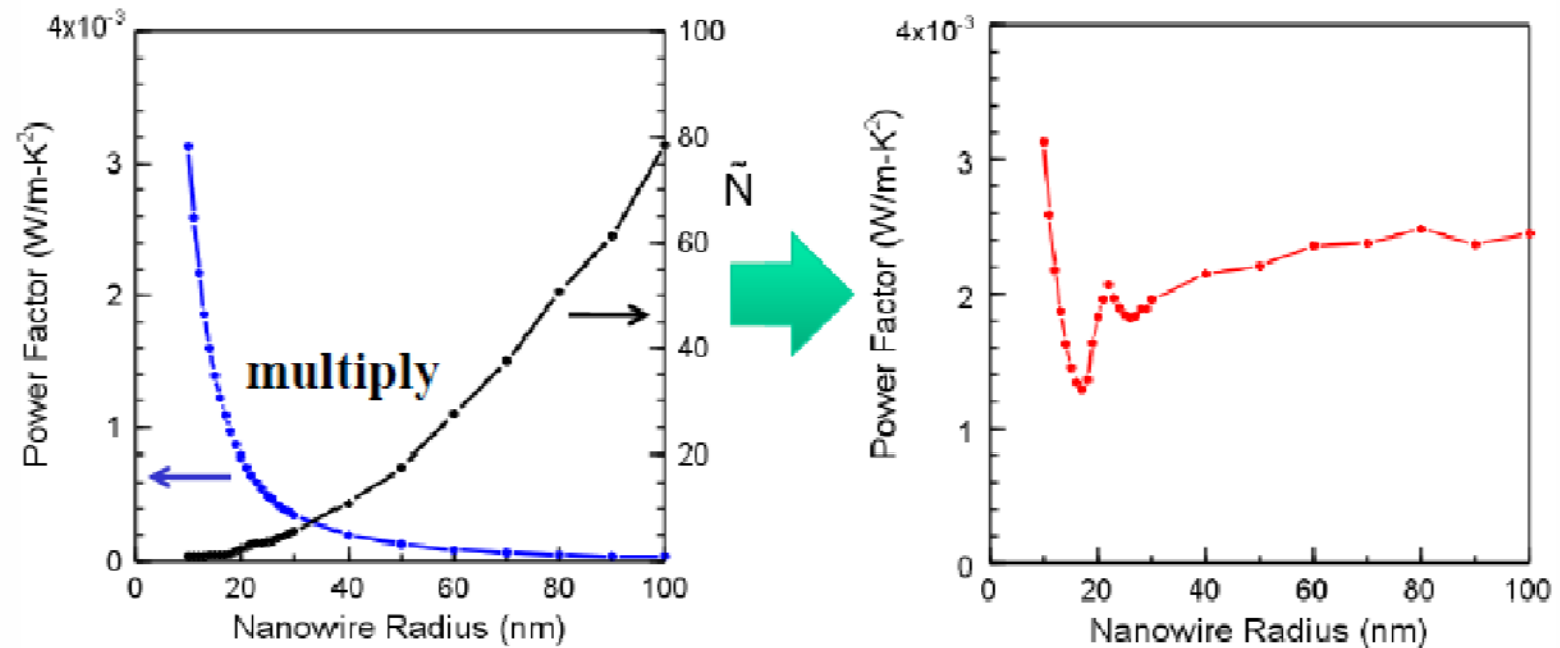
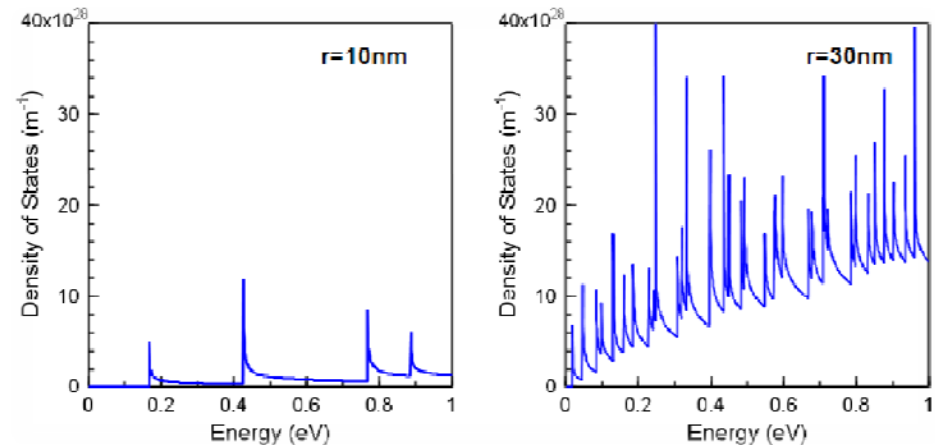
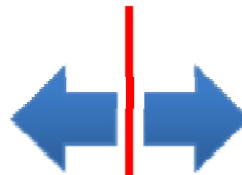


Figure 1. Electronic density of states for a) a bulk 3D crystalline semiconductor, b) a 2D quantum well, c) a 1D nanowire or nanotube, and d) a 0D quantum dot. Materials systems with low dimensionality also exhibit physical phenomena, other than a high density of electronic states (DOS), that may be useful for enhancing thermoelectric performance (see text).



$$PF(r) = PF(r)_{\text{one subband}} \times \text{Numbers of Subbands}(r)$$

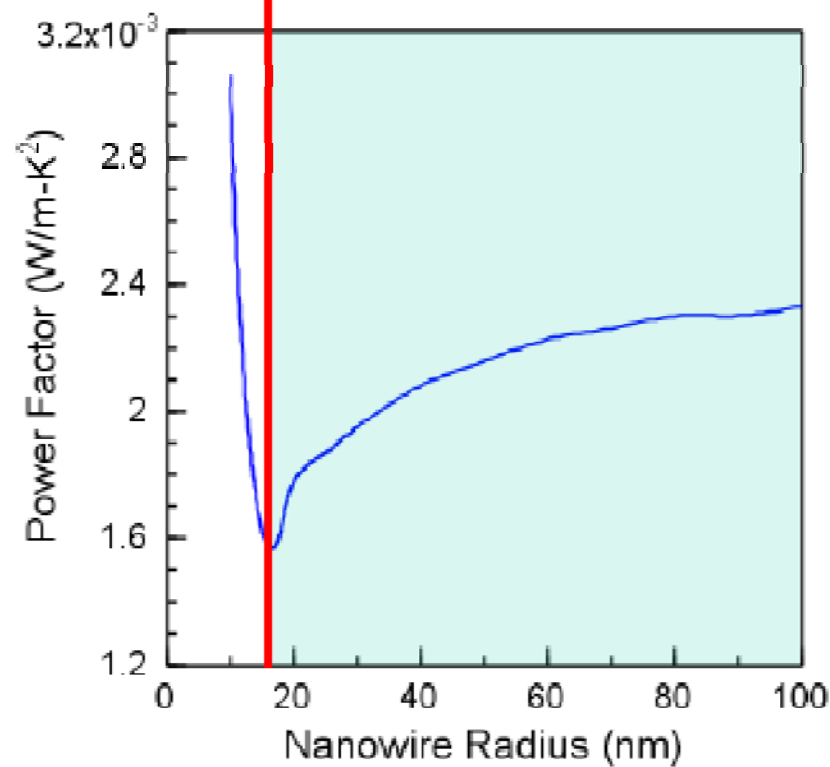
*Strongly Confined
System*



*Weakly Confined
System*

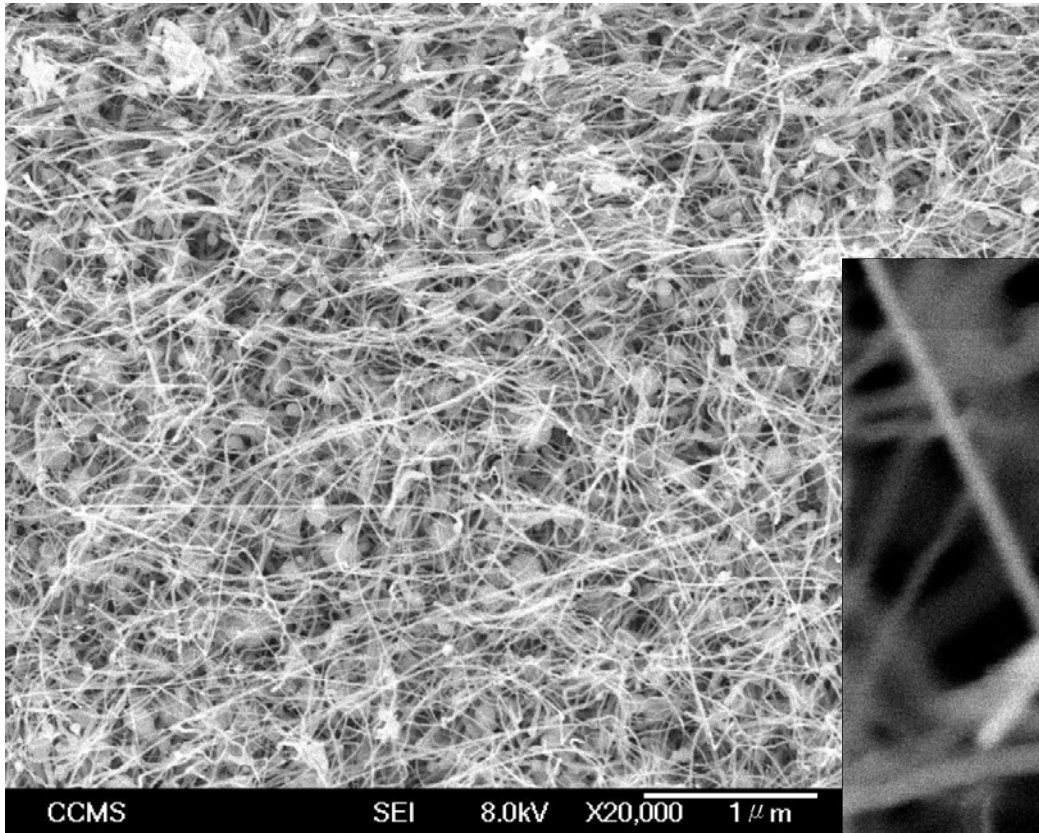
"DRESSELHAUS REGIME"

$$PF \propto r^{-2}$$

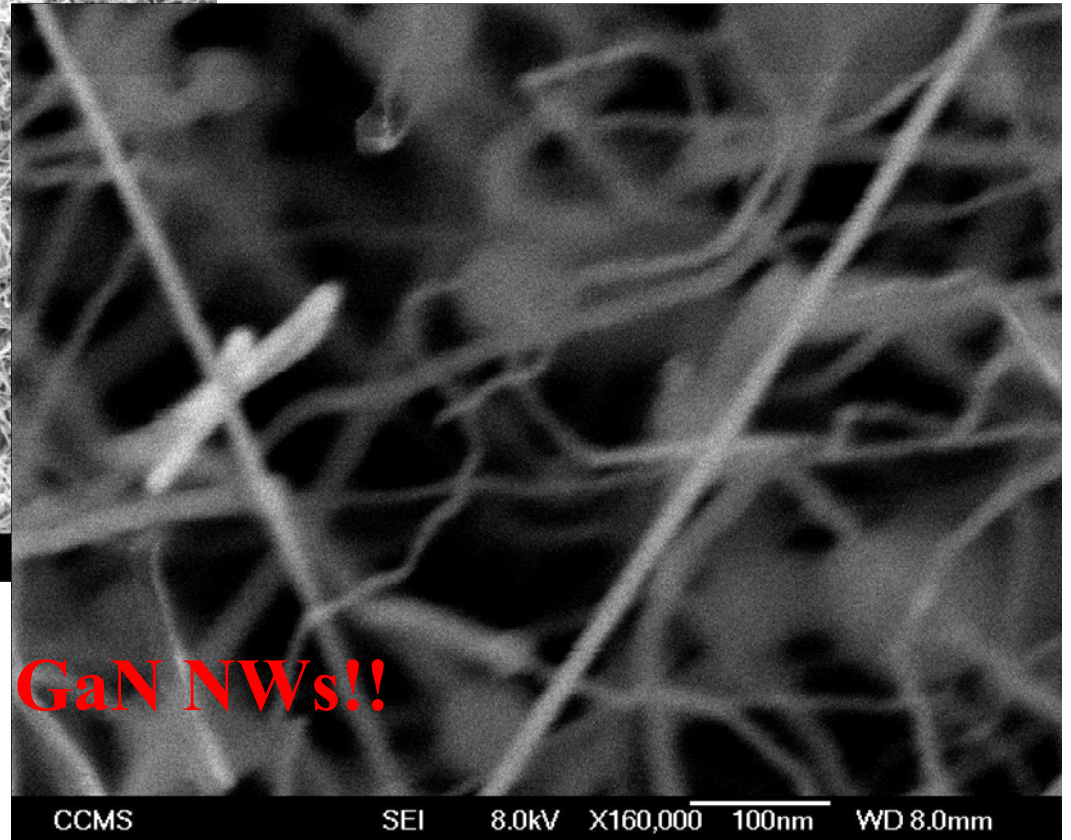


**Degeneracy
dominates!**

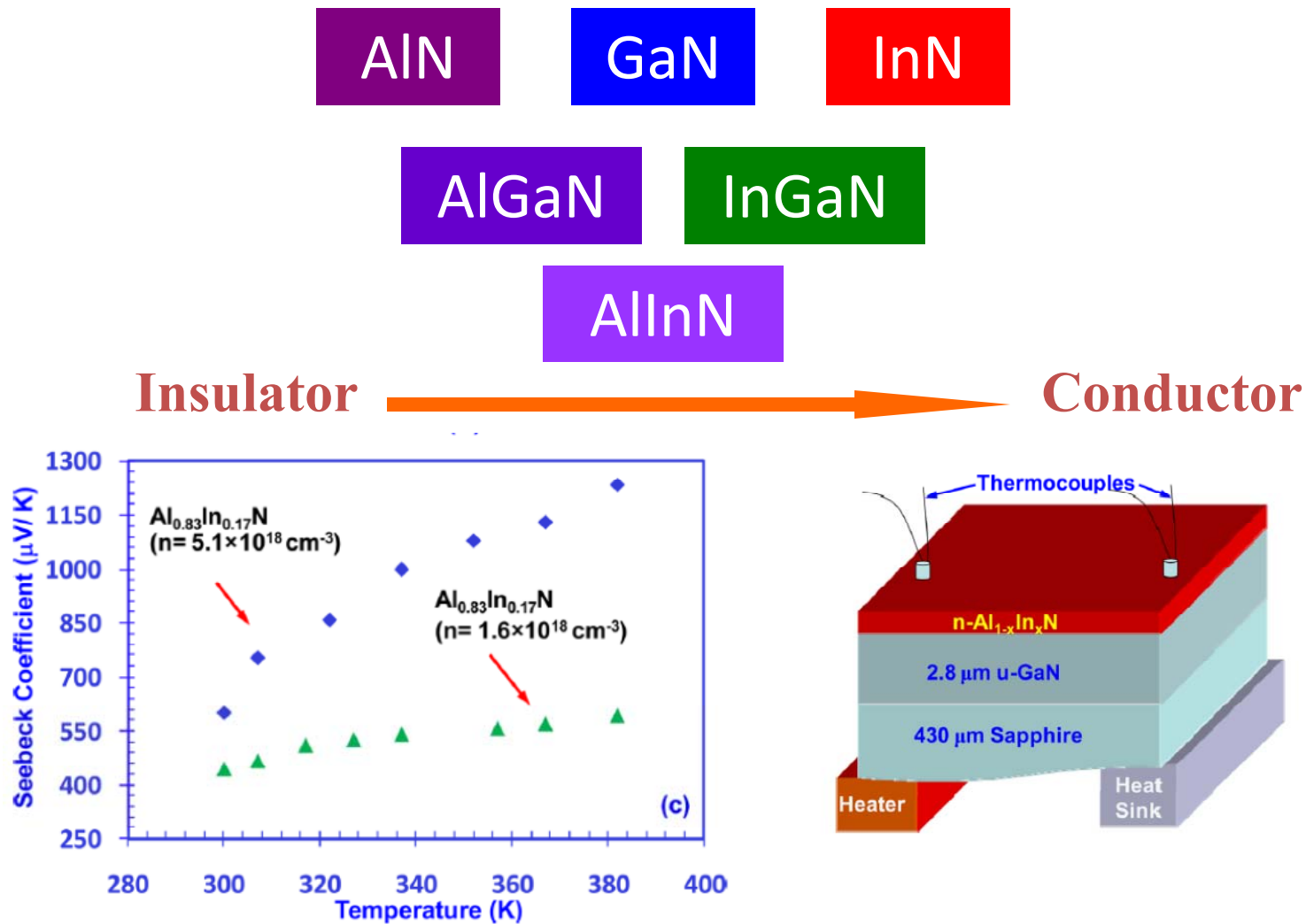
Ultra-thin GaN Nanowires grown by MOCVD



10 μ m long & 10nm wide GaN NWs!!



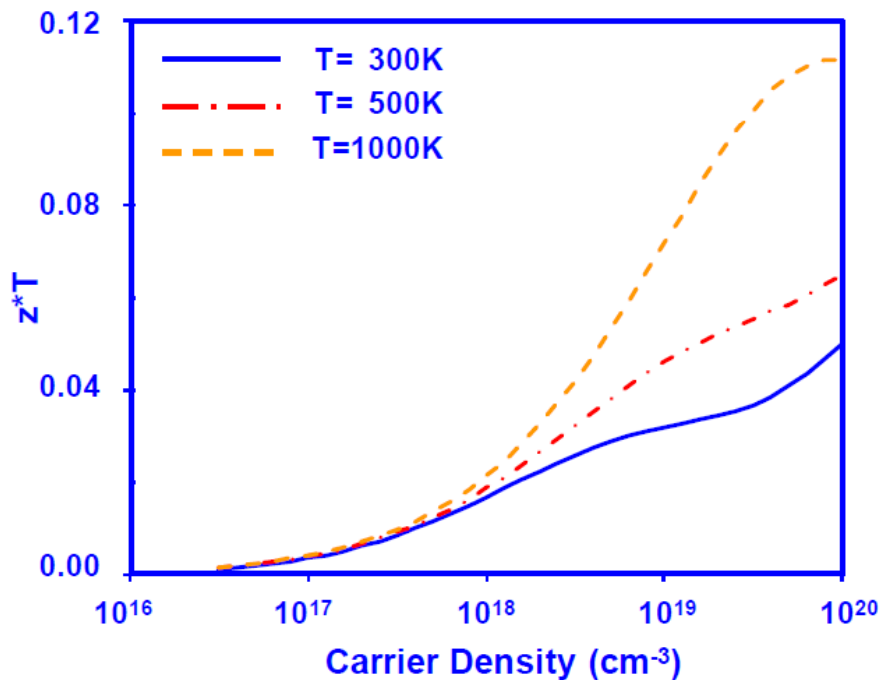
Nitride-Based Thermoelectric Materials



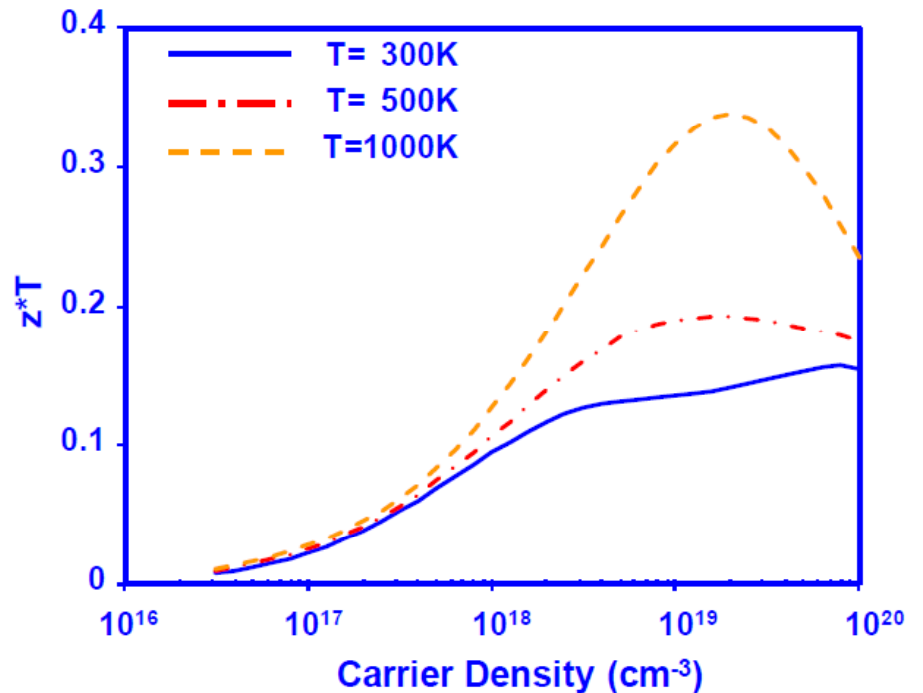
III-Nitride groups are expected to be good TE materials with high Seebeck coefficient or high power factor.

J. Zhang *et al.* JAP 110,043710 (2011)

ZT of III-V ternary (Simulation)

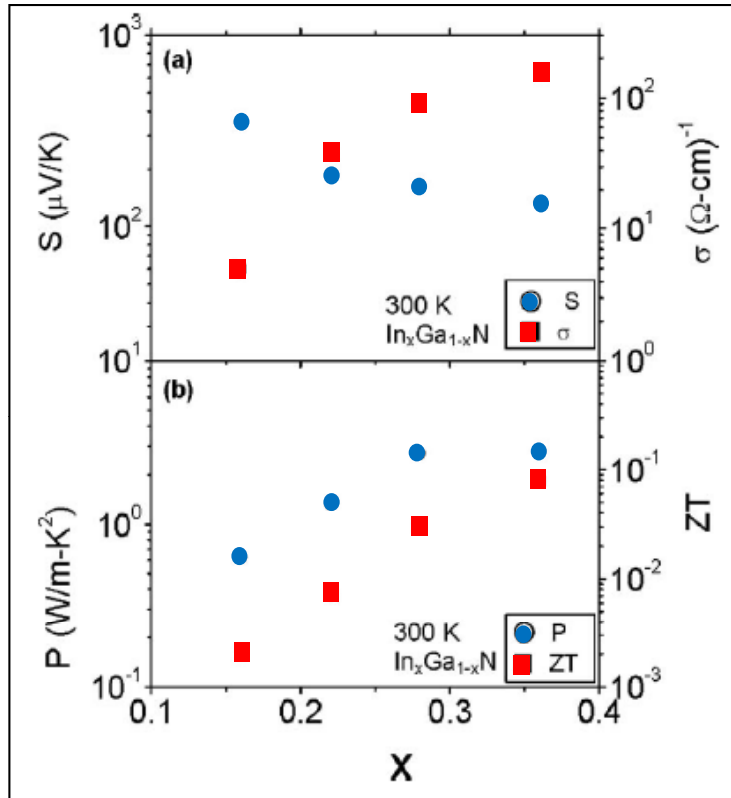


ZT of $\text{Al}_{0.4}\text{Ga}_{0.6}\text{N}$ as a function of carrier density at 300K (blue), 500 K (red), and 1000 K (yellow)

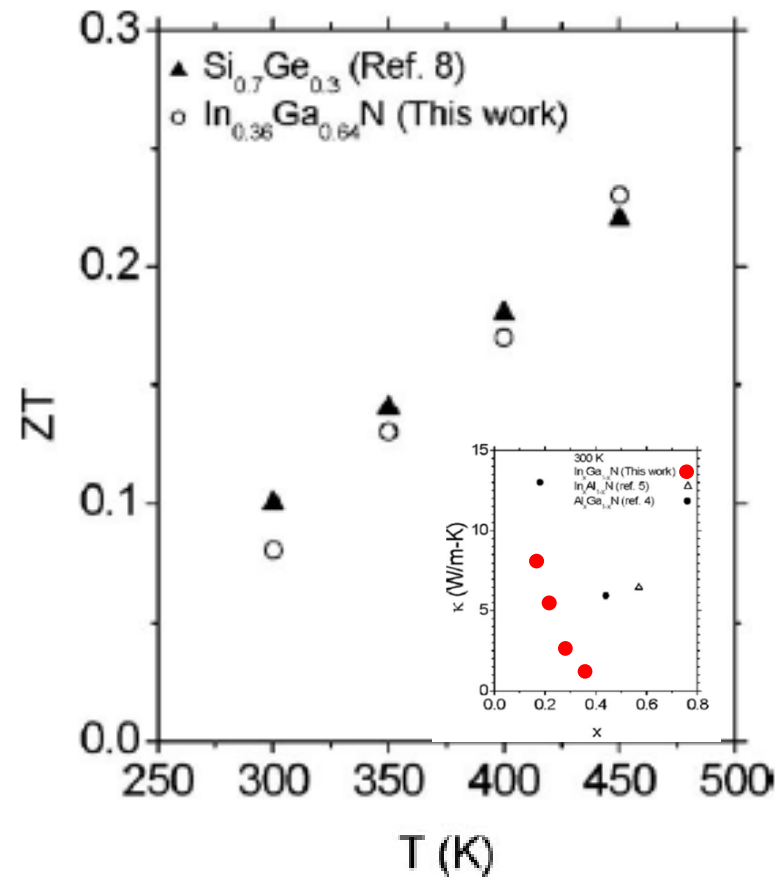


InGaN is expected to be the best TE material among III-nitride. This figure depicts ZT of $\text{In}_{0.45}\text{Ga}_{0.55}\text{N}$ as a function of carrier density at 300 K (blue), 500 K (red), and 1000 K (yellow)

ZT of InGaN ternary (Experiment)



(a) Seebeck coefficient S and electrical conductivity of $\text{In}_x\text{Ga}_{1-x}\text{N}$ alloys as functions of In content x . (b) Power factor $P=S^2\sigma$ and figure of merit ZT of $\text{In}_x\text{Ga}_{1-x}\text{N}$ alloys as functions of In content x measured at 300 K.



Measured ZT values of $\text{In}_{0.36}\text{Ga}_{0.64}\text{N}$ thin film from 300 to 450 K (110 nm)
 $ZT \sim 0.23$ was obtained.

Perspective



- Nanotechnology provides great opportunity for Thermoelectric materials.
- Demonstration of high ZT materials is yet to come.

Thermoelectric Effect

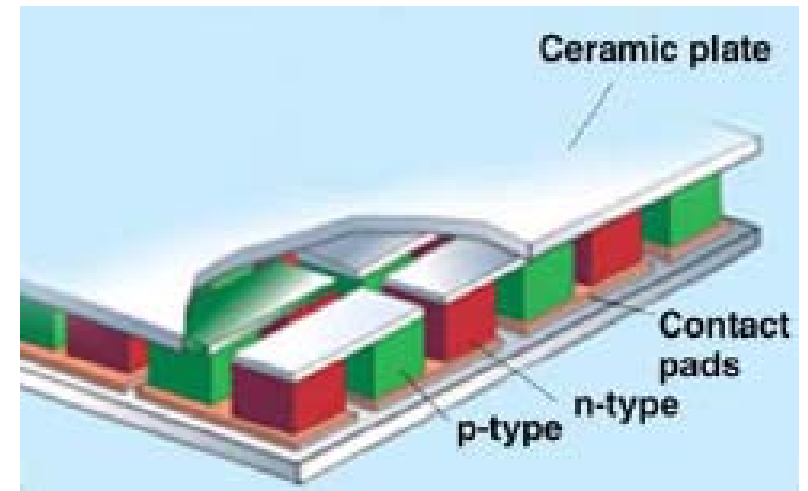
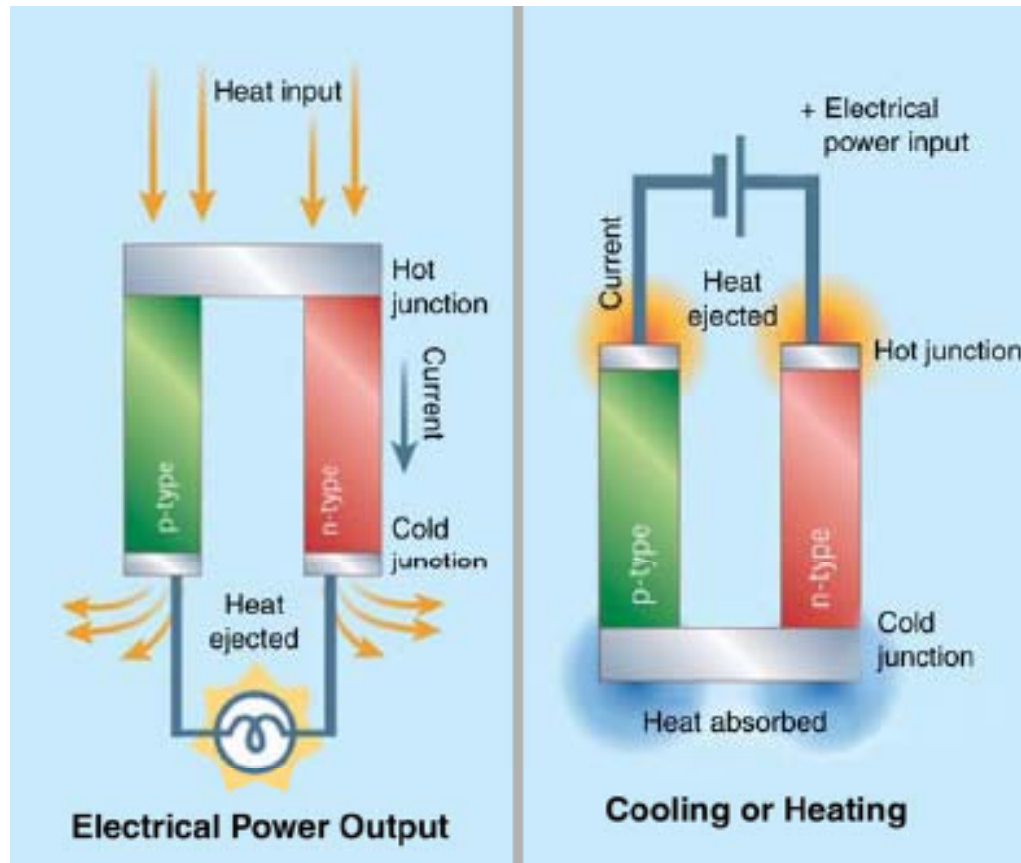


Figure of Merit & Efficiency

$$ZT = \frac{\alpha^2 \sigma T}{(\kappa_e + \kappa_{ph})}$$

$$\eta_{TE} = \eta_C \left(\frac{\sqrt{1 + ZT} - 1}{\sqrt{1 + ZT} + \frac{T_C}{T_H}} \right)$$

Carnot efficiency, $\eta_C = (T_H - T_C)/T_H$

