II.

Everyone did (a), (b), (c) correctly so I shall just add a few remarks.

(a) Equation of motion

$$M \frac{d^2 u_n}{dt^2} = K(u_{n+1} + u_{n-1} - 2u_n) + K_2(u_{n+2} + u_{n-2} - 2u_n) + \cdots \quad \cdots (*)$$

$$u_{n+1}/u_n = e^{ika}$$

$\therefore$ for small k,

$$u_{n+1} + u_{n-1} - 2u_n = u_n(e^{ika} + e^{-ika} - 2)$$

$$= 2u_n(\cos ka - 1) = -4u_n \sin^2\frac{ka}{2}$$

$$\to -u_n(ka)^2$$

for further neighbors

$$u_{n+j} + u_{n-j} - 2u_n = u_n(e^{ikja} + e^{-ikja} - 2)$$

$$\simeq u_n\Big[1 + ikja + \frac{(ikja)^2}{2!} + 1 - ikja + \frac{(-ikja)^2}{2!} - 2\Big)$$

$$\propto u_n k^2$$

$\therefore$ must have $\omega^2 \propto k^2$

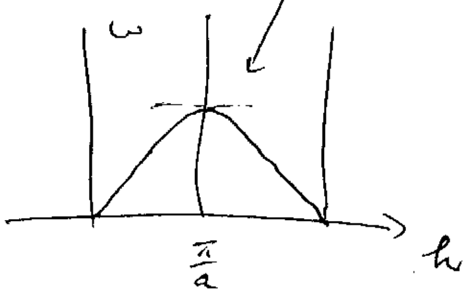
(b) group velocity at $k = \pm\frac{\pi}{a}$

$$M\omega^2 = \quad \cdots \quad \cos(jka)$$

$$\frac{d\omega}{dk} \propto \quad \cdots \quad \sin(jka) = 0 \quad \text{at } k = \pm\frac{\pi}{a}$$

<u>OR</u>: notice that $\omega(k) \overset{(1)}{=} \omega(k + \frac{2\pi}{a})$, $\omega(k) \overset{(2)}{=} \omega(-k)$

$\overset{*}{\Rightarrow}$ $\omega(\frac{\pi}{a} + k') = \omega(\frac{\pi}{a} - k') \Rightarrow$ slope of ω vs k at $\frac{\pi}{a}$ must be zero



(curve symmetric about $k = \frac{\pi}{a}$)

Same argument at $k = -\frac{\pi}{a}$.

(c) $k \to k + \frac{2\pi}{a}$ ① does not affect $\frac{u_{n+1}}{u_n} = e^{ika}$

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* Proof $\omega(\frac{\pi}{a} + k') \overset{(2)}{=} \omega(-\frac{\pi}{a} - k') \overset{(1)}{=} \omega(\frac{\pi}{a} - k')$